Optimisation Framework for Rail Traffic Control at a Single Junction

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Outline

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3. Optimisation Models
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   - Sequence optimisation
4. Case Study: Edgware Road Station
5. Conclusion
Introduction

RTS
- Real-time traffic management
- High capacity, energy-efficient, on-time

Strategy
- Right place, right time and right speed

Method
- Use of real-time data on train position and speed
- Dynamic optimisation of train movement
**Example: Edgware Road Station**

- Sequence at the bottleneck section
- Trajectory control on approaches from \(A_1\) to \(B\)
- Improve capacity
- Consider energy usage
- Dynamic controls
- Speed control at timing points
- Optimisation framework

From: High St Kensington

To: Hammersmith
Optimisation Framework

- Integrate energy optimisation with real-time rescheduling
- Optimise train sequence at junctions
- Optimise speed profiles within sections
Optimisation Framework

Schedule → Control centre → Sequence Optimisation → System Solution
Optimisation Framework

- Schedule
- Control centre
- Sequence Optimisation
- Individual trains
- Speed Profile Control
- System Solution
Optimisation Framework

Outputs of Sequence Optimisation
1) Signal timing/earliest entry time
2) Speed requirement

Outputs of Trajectory Optimisation
1) Optimised speed profiles
2) Time/speed anticipations
Plan movement of each train to specified point, arriving at

- Right position
- Right time
- Right speed

Achieve exit speed to limit further delays downstream

Manage energy usage

Consider:

- Tractive force available
- Resistive forces
  (both of which depend on train speed)
- Line speed limit
Model

• Only tractive force is supplied by train engine
• Continuous acceleration
• Minimise mechanical energy
• Newton’s second law of motion
• Boundary conditions: time, position, speed
• Other constraints: Acceleration/braking capabilities
Mathematical model

Minimise energy (without considering gradient)

\[
\min E = \int_{0}^{T} u_{tr}(t)v_{t}dt
\]

Subject to

\[
\dot{v} = u_{tr}(t) - u_{b}(t) - F_{R}(v_{t})
\]
\[
\dot{x} = v_{t}
\]

The boundary conditions of this problem are

\[
v(0) = v_{0}, v(T) = v_{T}
\]
\[
x(0) = x_{0}, x(T) = x_{T}
\]
\[
0 \leq u_{tr}(v) \leq g_{tr}(v)
\]
\[
0 \leq u_{b}(v) \leq g_{b}(v)
\]
Optimality of this model

Pontryagin’s minimum principle: five optimal regimes

- Traction regime
- Cruising regime
- Coasting regime
- Stabilization regime
- Braking regime
Solution approach: heuristic control

a. $v_{exit} < v_T(coasting)$

b. $v_{exit} > v_T(coasting)$
Sequence Optimisation

Sequence of trains at the junction is optimised

Plan sequence to minimise:
  • knock-on delay of trains
  • over short-term future

Combine with trajectory optimisation
  • blocking time model
  • alternative graph
Alternative Graph

$p_{ik}$: Running time of train $i$ on section $k$

$t_{ik+1} \geq t_{ik} + p_{ik}$
Alternative Graph

Train A

19 → 20 → 21

$\mathbf{a}_{AB21}$

Training B

29 → 20 → 5

$\mathbf{a}_{BA5}$

$p_{ik}$: Running time of train $i$ on section $k$

$t_{ik+1} \geq t_{ik} + p_{ik}$

$a_{ijk}$: Clearing, switching time of train $i$ on section $k$, plus sight and reaction time of train $j$ when approaching section $q$

$t_{ik} \geq t_{jq} + a_{ijk}$
Solution approach

- Sequence solution: heuristic method
- Check time feasibility between consecutive trains at each block section using blocking time model
- Update speed and time at boundaries of block section
  - Trajectory optimisation
  - Current position/speed of the train
  - Earliest clearing time of downstream signal
  - Scheduled speed profile as reference
Edgware Road Station

- Change times of signal and point status are considered
- 14 H&C line services, 15 Circle/District line services
- Dwell times at station are constant
- Comparison of speed coordination process with/without trajectory optimisation
- Various initial headways
Speed control strategy

Track sections from Paddington Station to Edgware Road Station

- Speed profile with optimal speed control
  - Scheduled Speed Profile
  - Earliness = 6s
  - Earliness = 2s
  - Earliness = 4s
  - Earliness = 0s
  - Earliness = 2s
  - Earliness = 4s
- Speed profile without optimal speed control
  - Earliness = 0s
  - Earliness = 2s
  - Earliness = 4s
  - Earliness = 6s
  - Earliness = 10s
  - Speed Limit
## Performance results

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Hierarchical Optimisation with Trajectory Optimisation</th>
<th>Variable Speed Conflict Resolution without Trajectory Optimisation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma$ (s)</td>
<td>Max Delay (s)</td>
<td>Mean Delay (s)</td>
</tr>
<tr>
<td>0</td>
<td>2.46</td>
<td>2.04</td>
</tr>
<tr>
<td>30</td>
<td>19.31</td>
<td>3.60</td>
</tr>
<tr>
<td>60</td>
<td>23.81</td>
<td>3.31</td>
</tr>
<tr>
<td>90</td>
<td>49.65</td>
<td>7.10</td>
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<tr>
<td>120</td>
<td>103.26</td>
<td>17.48</td>
</tr>
</tbody>
</table>

- Knock-on delay: 1~2 seconds saved
- Energy consumption: 2.9% ~4.5% saved
- Similar computational time
Benefits of optimisation

• Optimisation framework
  ➢ Effective response to perturbation
  ➢ Reduce delay propagation and knock-on delay
  ➢ Reduce energy usage

• Speed control
  ➢ Efficient for real-time traffic control
  ➢ Plan punctual train operations (time, speed)
Conclusion

Distributed optimisation (sequence and trajectory) improves performance

- Informed trajectory and sequence control can save:
  - Energy
  - Travel time
- Aids recovery after perturbation
- Capacity improvement on bottleneck section

Rapid calculation of speed profile can be applied on-line for real-time rescheduling
Thank You!

Q&A