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# Optimisation Framework for Rail Traffic Control at a Single Junction

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# Outline

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3. Optimisation Models
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   - Sequence optimisation
4. Case Study: Edgware Road Station
5. Conclusion

# Introduction

**RTS**

- Real-time traffic management
- High capacity, energy-efficient, on-time

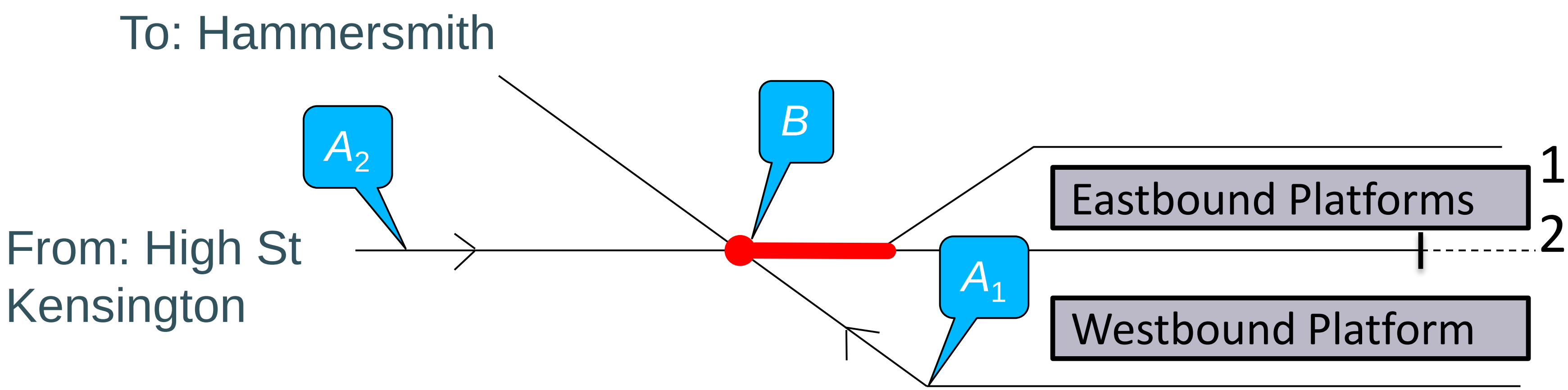
**Strategy**

- Right place, right time and right speed

**Method**

- Use of real-time data on train position and speed
- Dynamic optimisation of train movement

# **Example:** Edgware Road Station




- Sequence at the bottleneck section
- Trajectory control on approaches from $A_i$ to $B$

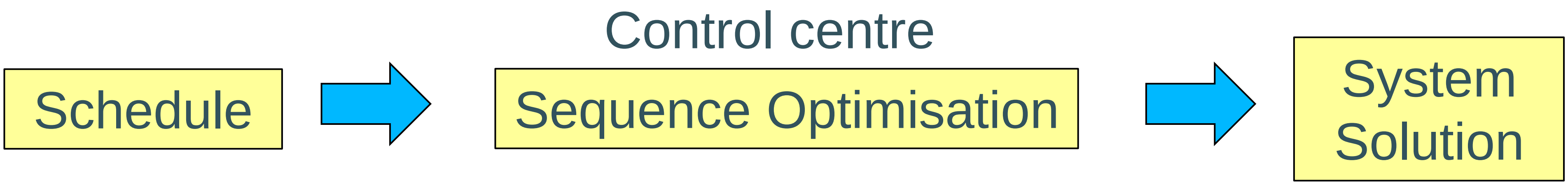
- Improve capacity
- Consider energy usage
- Dynamic controls

- Speed control at timing points
- Optimisation framework

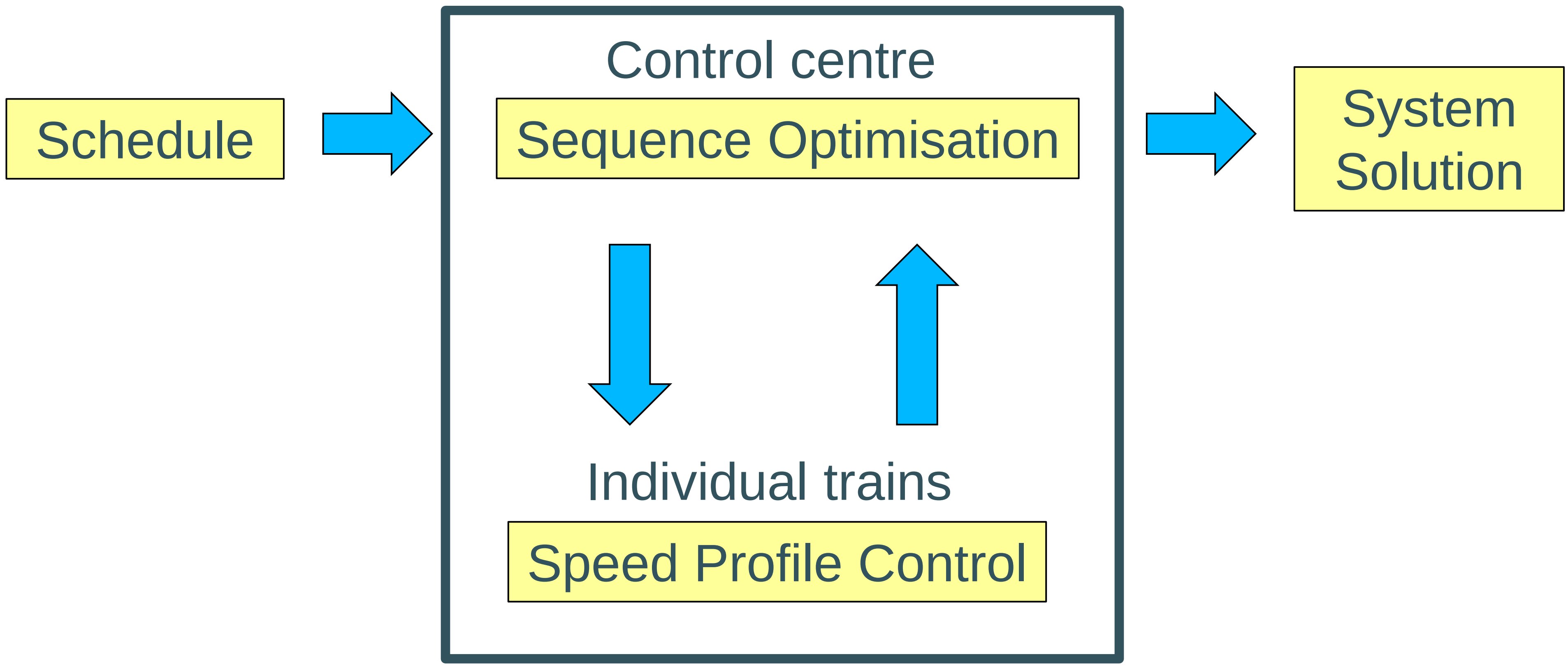
# Optimisation Framework

- Integrate energy optimisation with real-time rescheduling
- Optimise train sequence at junctions
- Optimise speed profiles within sections

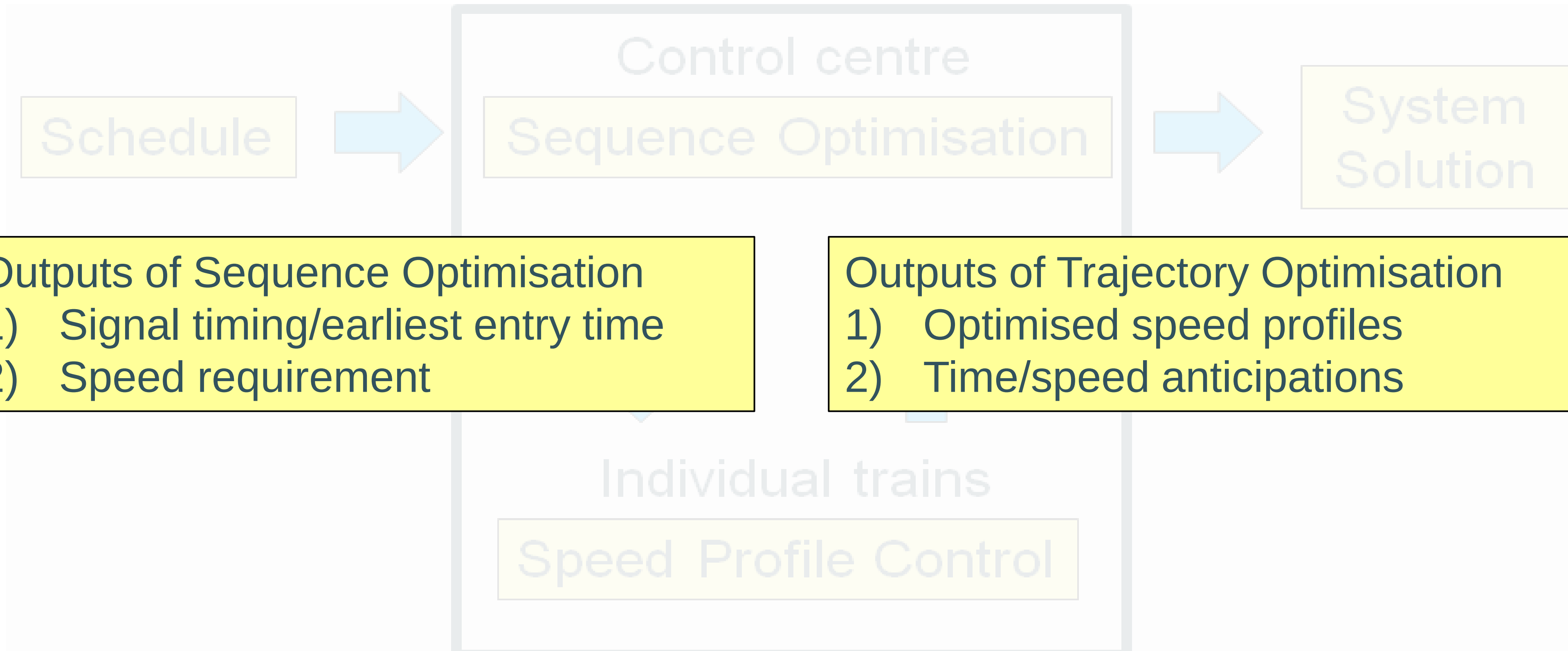
# Optimisation Framework



# Optimisation Framework



# Optimisation Framework




Outputs of Sequence Optimisation
1) Signal timing/earliest entry time
2) Speed requirement

Outputs of Trajectory Optimisation
1) Optimised speed profiles
2) Time/speed anticipations

# Trajectory Optimisation

Plan movement of each train to specified point, arriving at

- Right position
- Right time
- Right speed

Achieve exit speed to limit further delays downstream

Manage energy usage

Consider:

- Tractive force available
- Resistive forces

(both of which depend on train speed)

- Line speed limit

# Model

- Only tractive force is supplied by train engine
- Continuous acceleration
- Minimise mechanical energy
- Newton's second law of motion
- Boundary conditions: time, position, speed
- Other constraints: Acceleration/braking capabilities

# Mathematical model

Minimise energy (without considering gradient)

$$\min E = \int_0^T u_{tr}(t) v_t \, dt$$

Subject to

$$\dot{v} = u_{tr}(t) - u_b(t) - F_R(v_t)$$
$$\dot{x} = v_t$$

The boundary conditions of this problem are

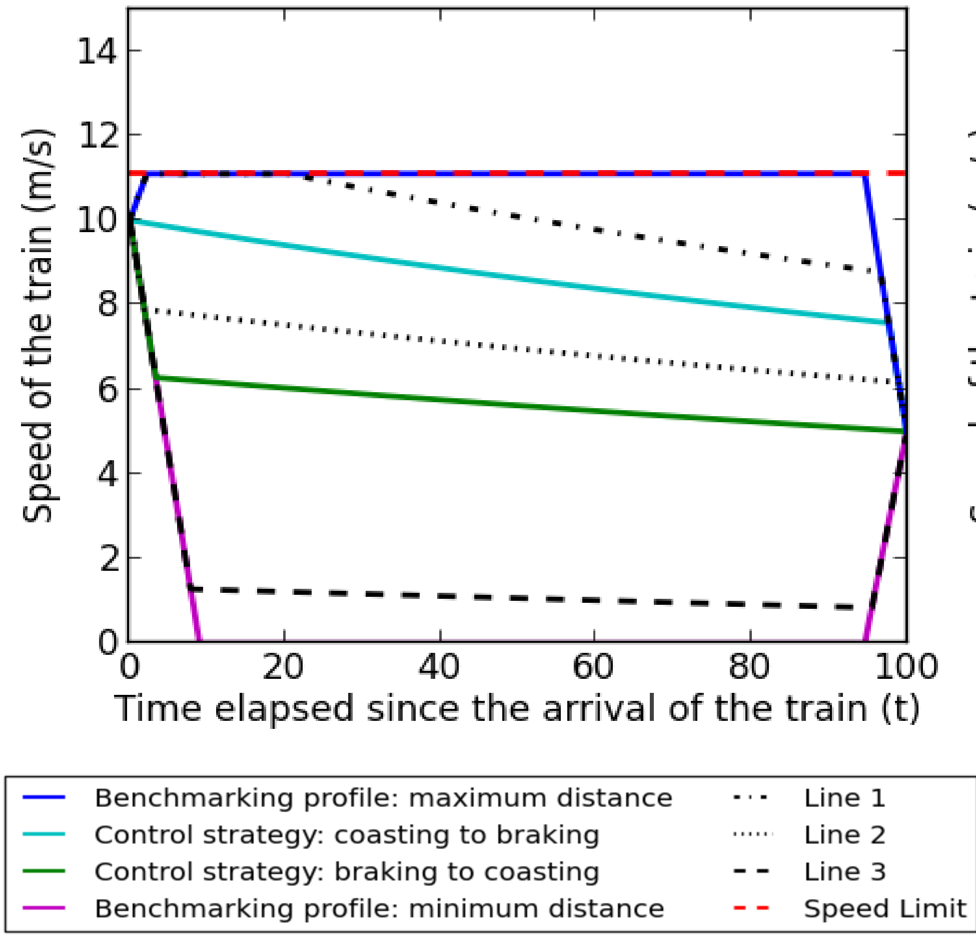
$$v(0) = v_0, v(T) = v_T$$
$$x(0) = x_0, x(T) = x_T$$
$$0 \leq u_{tr}(v) \leq g_{tr}(v)$$
$$0 \leq u_b(v) \leq g_b(v)$$

# Optimality of this model

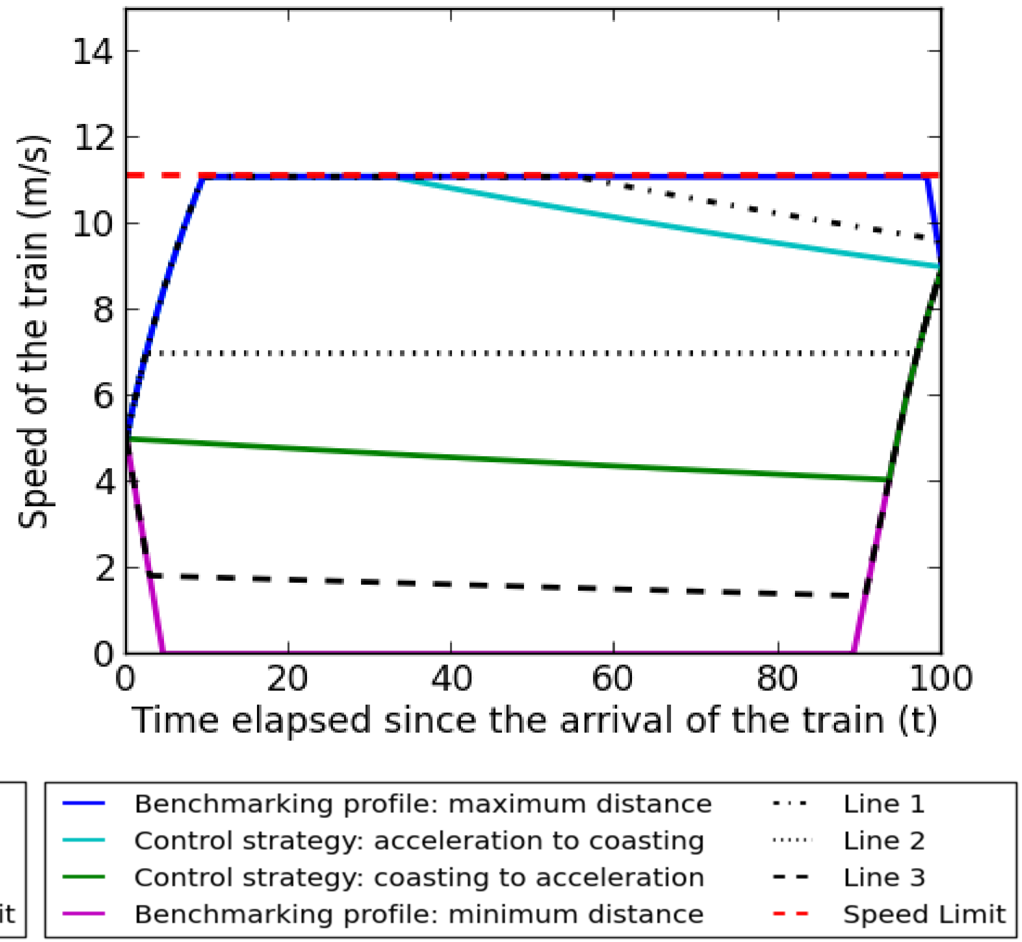
## Pontryagin's minimum principle: five optimal regimes

- Traction regime
- Cruising regime
- Coasting regime
- Stabilization regime
- Braking regime

# Solution approach: heuristic control




a. $v_{exit} < v_T(coasting)$




b. $v_{exit} > v_T(coasting)$

# Sequence Optimisation

Sequence of trains at the junction is optimised

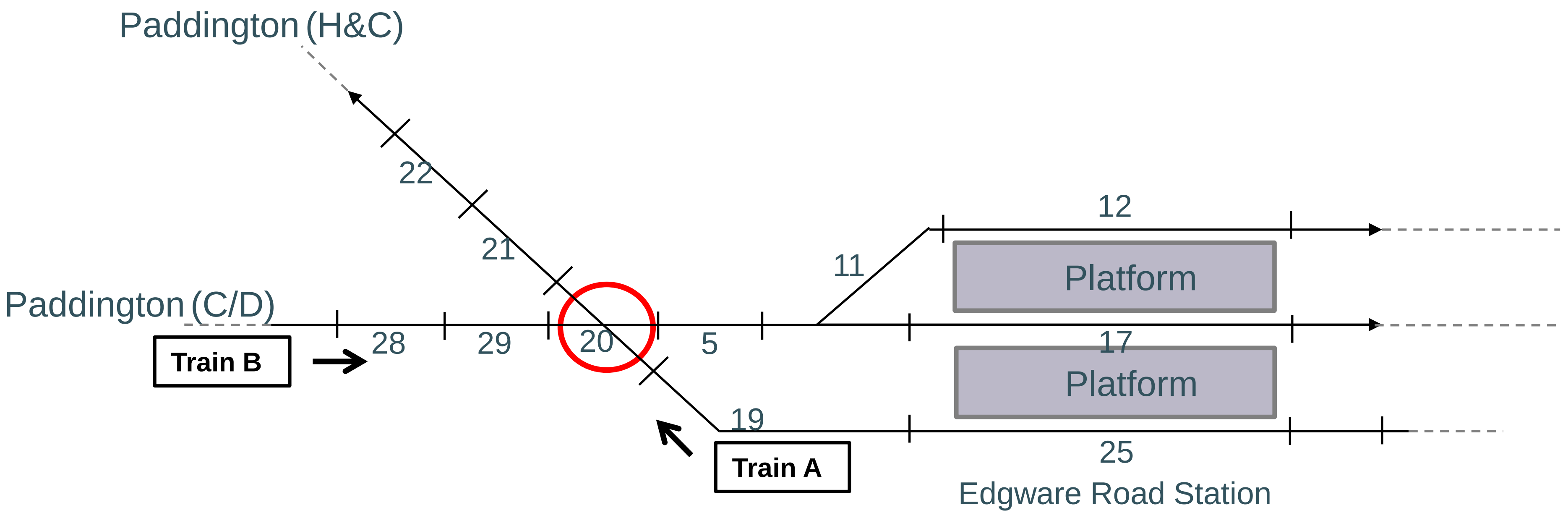
Plan sequence to minimise:

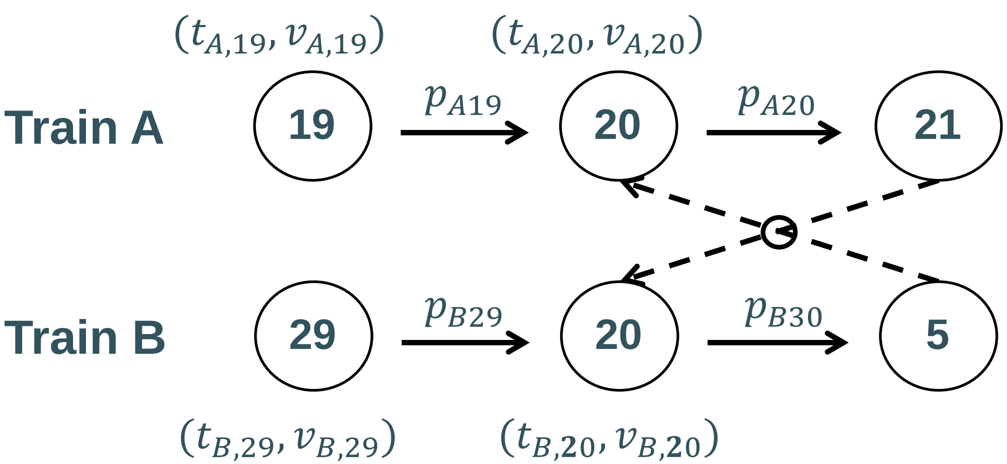
- knock-on delay of trains
- over short-term future

Combine with trajectory optimisation

- blocking time model
- alternative graph

# Alternative Graph

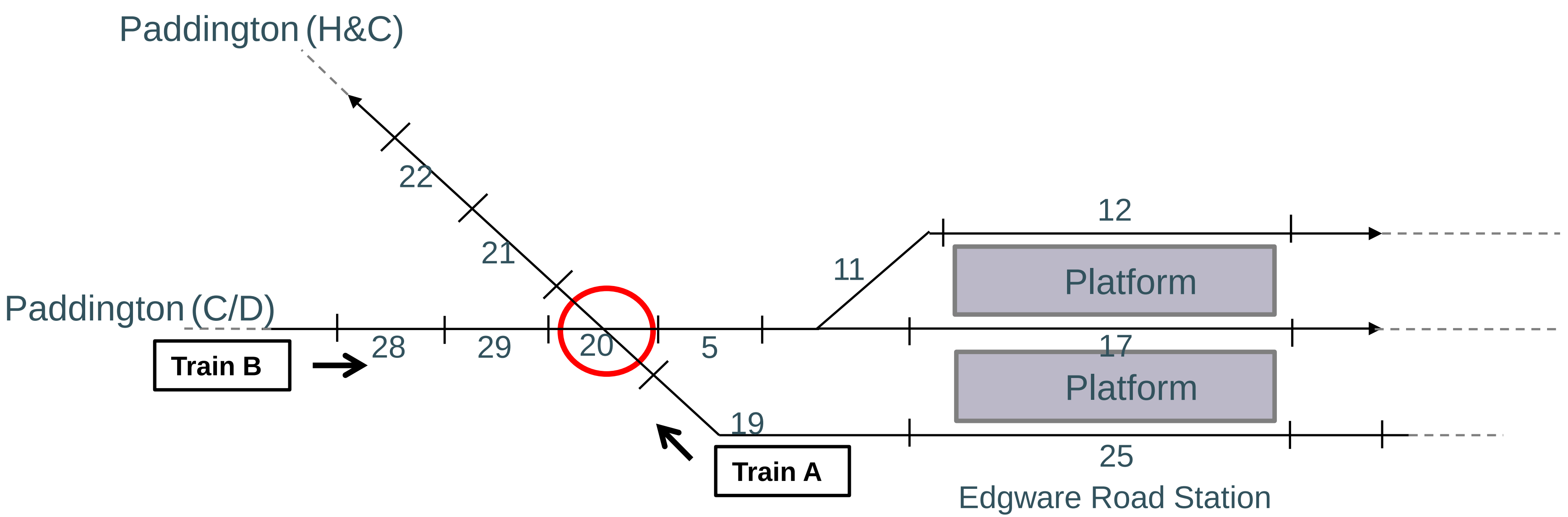


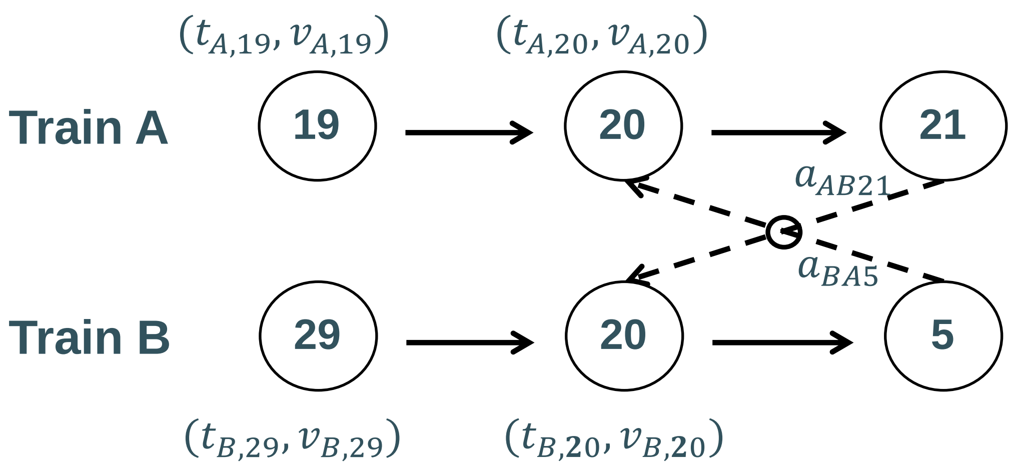




$p_{ik}$: Running time of train i on section $k$

$$t_{ik+1} \geq t_{ik} + p_{ik}$$

# Alternative Graph




$p_{ik}$: Running time of train i on section $k$

$$t_{ik+1} \geq t_{ik} + p_{ik}$$

$a_{ijk}$: Clearing, switching time of train $i$ on section $k$, plus sight and reaction time of train $j$ when approaching section $q$

$$t_{ik} \geq t_{jq} + a_{ijk}$$

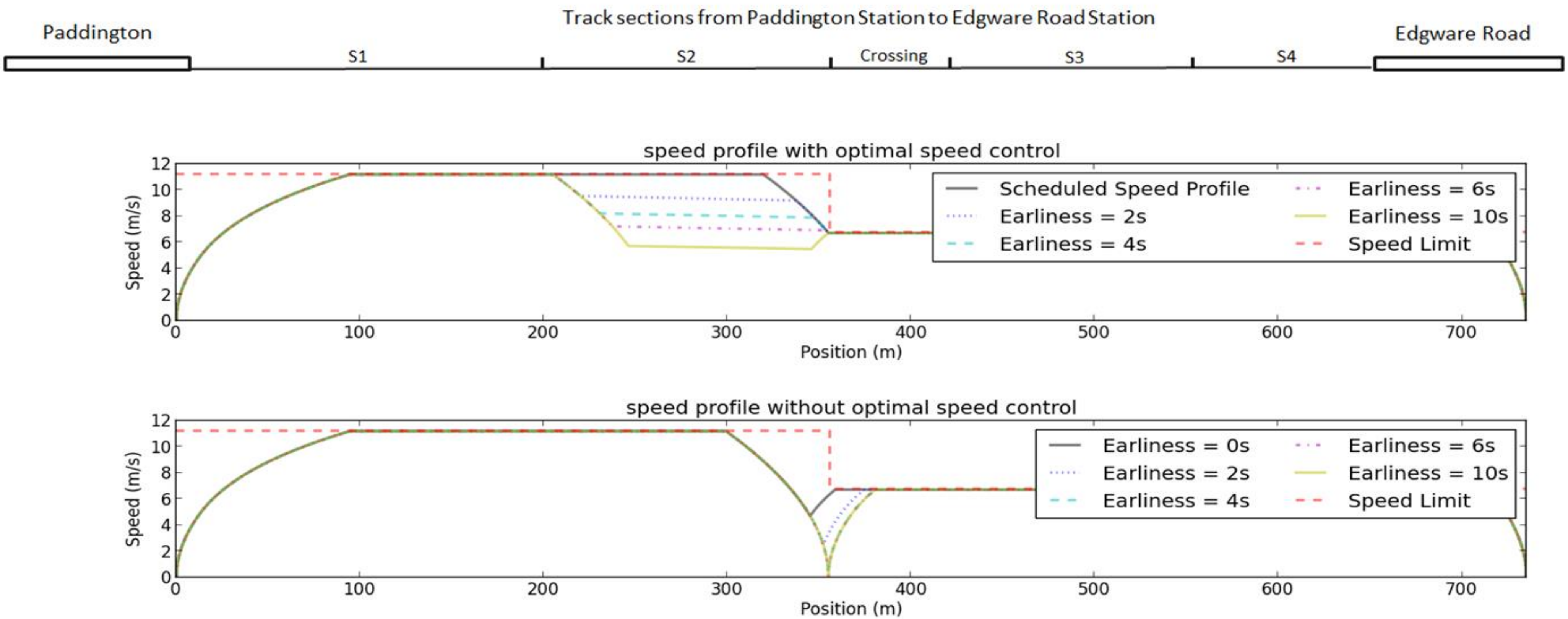
# Solution approach

- Sequence solution: heuristic method
- Check time feasibility between consecutive trains at each block section using blocking time model
- Update speed and time at boundaries of block section
  - Trajectory optimisation
  - Current position/speed of the train
  - Earliest clearing time of downstream signal
  - Scheduled speed profile as reference

# Edgware Road Station

- Change times of signal and point status are considered
- 14 H&C line services, 15 Circle/District line services
- Dwell times at station are constant
- Comparison of speed coordination process with/without trajectory optimisation
- Various initial headways

# Speed control strategy



# Performance results

| Scenario | Hierarchical Optimisation with Trajectory Optimisation | | | Variable Speed Conflict Resolution without Trajectory Optimisation | | |
|---|---|---|---|---|---|---|
| $\sigma$ $(s)$ | Max Delay (s) | Mean Delay (s) | Total Energy Consumption (kWh) | Max Delay (s) | Mean Delay (s) | Total Energy Consumption (kWh) |
| **0** | 2.46 | 2.04 | 135.94 | 4.55 | 3.76 | 148.92 |
| **30** | 19.31 | 3.60 | 138.51 | 24.48 | 5.08 | 145.12 |
| **60** | 23.81 | 3.31 | 139.68 | 27.64 | 4.41 | 144.20 |
| **90** | 49.65 | 7.10 | 140.19 | 52.63 | 8.34 | 144.83 |
| **120** | 103.26 | 17.48 | 142.18 | 107.26 | 19.39 | 146.43 |

- Knock-on delay: 1~2 seconds saved
- Energy consumption: 2.9% ~4.5% saved
- Similar computational time

# Benefits of optimisation

- Optimisation framework
  - Effective response to perturbation
  - Reduce delay propagation and knock-on delay
  - Reduce energy usage
- Speed control
  - Efficient for real-time traffic control
  - Plan punctual train operations (time, speed)

# Conclusion

Distributed optimisation (sequence and trajectory) improves performance

- Informed trajectory and sequence control can save:
  - Energy
  - Travel time
- Aids recovery after perturbation
- Capacity improvement on bottleneck section

Rapid calculation of speed profile can be applied on-line for real-time rescheduling

# Thank You!

# Q&A