Integrated Optimization Model for Train Scheduling and Utilization Planning in Mass Rapid Transit Systems

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Operational Planning Process for a Rail System

**Input**
- Marketing research, demand modeling, historical ticketing
- Train travel time for each stopping pattern
- Line capacity, train capacity, costs, etc.
- Headway, train running time, etc.
- Fleet size, round trip time, Depot & station capacity, leave plan, headway, etc.

**Planning Activity**
- Travel demand analysis
- Train stop planning
- Train service planning
- Train scheduling
- Resource planning (trainset, crew, track, etc)
- Track utilization plan

**Output**
- Origin-destination (O-D) matrix
- Combination of stopping patterns
- Expected timetable
- Timetable
- Train Utilization plan
- Crew utilization plan

Scope of the study
Train Scheduling Process

- Determine the actual schedule with the least difference with the expected timetable and no conflict
- Satisfy the constraints on running time, dwell time, headway between adjacent trains, and station capacity, etc.

Expected timetable

Feasible timetable (conflict free)
Methods on Train Scheduling

Heuristic method

- Start from an initial solution, gradually improve to good timetable
- Divide into sub-problem, individually solved by an expert system

Simulation method

- Event-driven simulation model, continuously updated and executed until a terminating condition
- Generally lack capability to optimize timetables, time-consuming

Mathematical programming method

- Linear or nonlinear structure
- Many commercial software can help to solve
Train Utilization Planning

• Arrange the connections among various train trips to cover all trains in the timetable

• Satisfy the constraints on preparation time, turnaround time, terminal station capacity, depot capacity, etc
Methods on Train Utilization Planning

• Assignment model aims to use the least number of rolling stock to cover a given timetable
• Other objectives may include delay propagation, operational cost, etc.

Past studies usually deal with scheduling and train utilization planning separately
Motivation to Integrate Scheduling & Utilization Planning

• The processes that produce timetables and train utilization plans are typically performed individually by different departments.

• These two processes are however highly dependent on each other.

• A feasible and efficient timetable may end up with an inefficient or even infeasible train utilization plan.
Integrated Optimization Model

Min \( Z = \sum_{i \in \Omega \cup \Psi} \alpha_i |D_i - \delta_i| + \sum_{i \in \Omega \cup \Psi} \beta_i (MA_i - MD_i - \tau_i) + \sum_{s \in S} \gamma_s TI_s \),

converted into a linear format

Min \( Z = \sum_{i \in \Omega \cup \Psi} \alpha_i E_i + \sum_{i \in \Omega \cup \Psi} \beta_i (MA_i - MD_i - \tau_i) + \sum_{s \in S} \gamma_s TI_s \),

\( E_i \geq D_i - \delta_i \quad i \in \Omega \cup \Psi, \)

\( E_i \geq \delta_i - D_i \quad i \in \Omega \cup \Psi, \)
Departure Time and Running Time Constraints

\[ D_i \geq LD_i \quad i \in \Omega \cup \Psi, \]
\[ D_i \leq UD_i \quad i \in \Omega \cup \Psi, \]

\[ A_i - D_i = \omega_i + \theta_i \quad i \in \Omega \cup \Psi, \]
\[ \text{between two stations} \]

\[ MA_i - ED_i \geq \theta_i^{td} - M \times E_i \quad i \in \Omega \cup \Psi, \]
\[ MA_i - ED_i \leq \theta_i^{td} + M \times E_i \quad i \in \Omega \cup \Psi, \]
\[ EA_i - MD_i \geq \theta_i^{tt} - M \times Y_i \quad i \in \Omega \cup \Psi, \]
\[ EA_i - MD_i \leq \theta_i^{tt} + M \times Y_i \quad i \in \Omega \cup \Psi, \]
\[ \text{between depot and terminal station} \]

\[ ED_i - A_i \geq \omega_{min}^r - M \times E_i \quad i \in \Omega \cup \Psi, \]
\[ ED_i - A_i \leq \omega_{max}^r + M \times E_i \quad i \in \Omega \cup \Psi, \]
\[ D_i - EA_i \geq \omega_{min}^p - M \times Y_i \quad i \in \Omega \cup \Psi, \]
\[ D_i - EA_i \leq \omega_{max}^p + M \times Y_i \quad i \in \Omega \cup \Psi, \]
\[ \text{before entering depot} \]
Headway Constraints

\[ A_j - A_i \geq h_{AA} - M \times (1 - SA_{i,j}) \quad i, j \in \Omega \text{ or } i, j \in \Psi, i \neq j, \]
\[ A_i - A_j \geq h_{AA} - M \times SA_{i,j} \quad i, j \in \Omega \text{ or } i, j \in \Psi, \]
i \neq j,

arrival-arrival

\[ D_j - D_i \geq h_{DD} - M \times (1 - SD_{i,j}) \quad i, j \in \Omega \text{ or } i, j \in \Psi, \]
i \neq j,
\[ D_i - D_j \geq h_{DD} - M \times SD_{i,j} \quad i, j \in \Omega \text{ or } i, j \in \Psi, \]
i \neq j,

departure-departure

\[ A_i - D_j \geq h_{DA} - M \times (1 - B_{i,j}) \quad i, j \in \Omega \text{ or } i, j \in \Psi, i \neq j, \]
\[ A_i - D_j \leq M \times B_{i,j} \quad i \in \Omega \land j \in \Psi \text{ or } i \in \Psi \land j \in \Omega, \]

arrival-departure

Headway constraints
Capacity Constraints

\[ \sum_{j \in \Omega} F_{i,j} + \sum_{j \in \Psi, j \neq i} S_{A,j,i} - \sum_{j \in \Omega} B_{i,j} - \sum_{j \in \Psi, j \neq i} Q_{i,j} \leq \eta_s - 1 \quad i \in \Psi, \]

\[ \sum_{j \in \Psi} F_{i,j} + \sum_{j \in \Omega, j \neq i} S_{A,j,i} - \sum_{j \in \Psi} B_{i,j} - \sum_{j \in \Omega, j \neq i} Q_{i,j} \leq \eta_s - 1 \quad i \in \Omega, \]

\[ TI^s + \sum_{j \in \Psi, j \neq i} M_{i,j} - \sum_{j \in \Omega} W_{i,j} - M \times E_i \leq \sigma_s - 1 \quad i \in \Psi, \]

\[ TI^s + \sum_{j \in \Omega, j \neq i} M_{i,j} - \sum_{j \in \Psi} W_{i,j} - M \times E_i \leq \sigma_s - 1 \quad i \in \Omega, \]
Terminal and Depot Operations Constraints

\[ \sum_{j \in \Omega} R_{i,j} + (1 - E_i) + EP_i = 1 \quad i \in \Psi, \]
\[ \sum_{j \in \Psi} R_{i,j} + (1 - E_i) + EP_i = 1 \quad i \in \Omega, \]
\[ \sum_{i \in \Psi} R_{i,j} + (1 - Y_j) + EY_j = 1 \quad j \in \Omega, \]
\[ \sum_{i \in \Omega} R_{i,j} + (1 - Y_j) + EY_j = 1 \quad j \in \Psi, \]

\[ E_i + Y_j \leq 2 + M \times (1 - R_{i,j}) \quad i \in \Psi \land j \in \Omega \text{ or } \]
\[ E_i + Y_j \geq 2 - M \times (1 - R_{i,j}) \quad i \in \Psi \land j \in \Omega \text{ or } \]
\[ i \in \Omega \land j \in \Psi. \]

\[ \sum_{j \in \Omega} MR_{i,j} + MP_i = (1 - E_i) \quad i \in \Psi, \]
\[ \sum_{j \in \Psi} MR_{i,j} + MP_i = (1 - E_i) \quad i \in \Omega, \]
\[ \sum_{i \in \Psi} MR_{i,j} + MY_i = (1 - Y_i) \quad i \in \Omega, \]
\[ \sum_{j \in \Omega} MR_{i,j} + MY_i = (1 - Y_i) \quad i \in \Psi. \]
Other Constraints

\[ \sum_{i \in \Psi} M Y_i \leq T I^s \]

\[ \sum_{i \in \Omega} M Y_i \leq T I^s \]

\[ D_j - A_i \geq \xi_{\text{min}} - M \times (1 - R_{i,j}) \quad i \in \Omega \land j \in \Psi \text{ or } i \in \Psi \land j \in \Omega, \]

\[ D_j - A_i \leq \xi_{\text{max}} + M \times (1 - R_{i,j}) \quad i \in \Omega \land j \in \Psi \text{ or } i \in \Psi \land j \in \Omega, \]

\[ MD_j - MA_i \geq \zeta_{\text{min}} - M \times (1 - MR_{i,j} + E_i + Y_j) \quad i \in \Omega \land j \in \Psi \text{ or } i \in \Psi \land j \in \Omega, \]
Case Study in Orange Line of the Kaohsiung Metro

14 km, 14 stations, 272 Trains/day

Sizihwan
Yanchengpu
City Council
Formosa Boulevard
Sinyi Elementary School
Cultural Center
Weiwuying
Fongshan West
Fongshan
Dadong
Fongshan Junior High School
Daliao
Depot

14 Tracks
Resulting Timetable

Only show a part of timetable (6:00~12:00)
# Resulting Utilization Plan

<table>
<thead>
<tr>
<th>Utilization path</th>
<th>Time(hour)</th>
<th>Mileage (km)</th>
<th>Service frequency (trips/day)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>06:07:52</td>
<td>512.734</td>
<td>39</td>
</tr>
<tr>
<td></td>
<td>06:24:15</td>
<td>187.612</td>
<td>14</td>
</tr>
<tr>
<td></td>
<td>06:34:15</td>
<td>500.588</td>
<td>38</td>
</tr>
<tr>
<td></td>
<td>06:44:07</td>
<td>389.368</td>
<td>28</td>
</tr>
<tr>
<td></td>
<td>06:57:53</td>
<td>266.344</td>
<td>20</td>
</tr>
<tr>
<td></td>
<td>07:03:01</td>
<td>54.440</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>07:13:01</td>
<td>474.344</td>
<td>36</td>
</tr>
<tr>
<td></td>
<td>07:17:52</td>
<td>278.490</td>
<td>21</td>
</tr>
<tr>
<td></td>
<td>07:22:52</td>
<td>343.124</td>
<td>26</td>
</tr>
<tr>
<td></td>
<td>07:38:01</td>
<td>343.124</td>
<td>26</td>
</tr>
<tr>
<td></td>
<td>07:48:01</td>
<td>343.124</td>
<td>26</td>
</tr>
</tbody>
</table>

**Sw**: Sizihwan Station  
**Dl**: Daliao Station  
**Dl_D**: Daliao Depot
## Comparison

<table>
<thead>
<tr>
<th>Weight</th>
<th>Solution method</th>
<th>Objective value (min)</th>
<th>Departure time disparity (min/train)</th>
<th>Fleet size (trains)</th>
<th>Solution time</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5:0.001:0.001 (500:1:1)</td>
<td>Current Timetable (Manual)</td>
<td>174.160</td>
<td>1.28</td>
<td>11</td>
<td>1 month</td>
</tr>
<tr>
<td></td>
<td>Integrated Model</td>
<td>97.991</td>
<td>0.72</td>
<td>11</td>
<td>24 hrs</td>
</tr>
</tbody>
</table>
Conclusions

• This model can determine optimal timetable, utilization plan, and fleet size simultaneously

• It significantly reduces solution time from one month to one day, and also improves the solution quality

• It can help railway companies automate the train scheduling and utilization planning processes with acceptable efficiency