Optimization of Life-Cycle Cost and Reliability Allocation for Rail Systems

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System design considers trade-off between cost and reliability

- A rail system consists of a number of subsystems, and each has its own cost and reliability.
- Planners have to carefully allocate the reliability and budget by examining the trade-off between cost and reliability.

### Rail System
- Reliability: 0.9
- Cost: 100 billion

### Subsystems
- **Infrastructure**
  - Reliability: 0.98
  - Cost: 50 billion
- **Rolling Stock**
  - Reliability: 0.97
  - Cost: 25 billion
- **Signal & Communication**
  - Reliability: 0.96
  - Cost: 15 billion
- **Electricity**
  - Reliability: 0.99
  - Cost: 10 billion
MTBF does not consider its impact to the customers

- Various reliability allocation methods have been developed in the past.
  - Weighting Method
  - Optimization Method

- For rail system, reliability (system reliability) can be defined as the mean time between failures (MTBF)

However, this attribute does not consider its effect on passengers (the consequence of failure)
System Reliability vs. Service Reliability

Failure frequency ➔ MTBF ➔ System Reliability

Consequence of failure ➔ Delay ➔ Service Reliability

Train failure
MTBF: 100,000 train-hour

1 minute delay (for one train)

Communication failure
MTBF: 100,000 train-hour

1 hour delay (for multiple trains)
Service reliability (e.g. delay or on-time percentage) is more favorable than system reliability because it considers customers’ satisfaction.

Planners should balance the trade-off among service reliability, system reliability, and LCC in Rail System Design.
Key Elements in Rail System Design – LCC

- LCC for railway systems typically includes capital investment, operating cost, and maintenance cost within the planning period.

\[
LCC = \text{Capital Cost} + \text{Operating Cost} + \text{Maintenance Cost}
\]

- Employing LCC is more appropriate in decision making than solely employing capital investment.
  - Some products have low capital investment but high operating and maintenance costs.
  - Others have high capital investment but low operating and maintenance costs.
System reliability is defined as MTBF or MDBF.

Relationship between system reliability and LCC can be obtained from suppliers.

Time or distance between failures:

Failure  Failure  Failure  Failure  Exposure
Service reliability identifies the effect on passengers.

Target Service Reliability – On-time arrival percentage (with no buffer): proportion of on-time operations in terms of total system operating time (in train-hour)

\[ r_{sys} = \left( P - \sum_{i=1}^{n} d_i \right) \times 100\% \]

- **On-time arrival percentage**
  - \( P \): Total system operational time (in train-hour) in a defined period
  - \( d_i \): Delay (in train-hour) of subsystem i
Optimization Framework

Investment Alternatives

Alternative Evaluator (AE)

Investment Alternatives Table

Minimize Cost Model

Maximize Reliability Model

Design Service Reliability

Investment Selector (IS)

Design System LCC

Output Data

- System Reliability
- Life Cycle Cost
- Service Reliability

Demand
Alternative Evaluator (AE)

- AE evaluates all possible alternatives and generates an investment alternatives table with their LCC, system reliability, and service reliability.
- Service reliability needs to be computed based on system reliability and operational data.

**Diagram:**
- **Investment Alternatives**
  - Alternative Evaluator (AE)
  - Demand
  - Investment Alternatives Table
  - **LCC and System Reliability can be obtained from suppliers**
  - **Operational data is provided by operators**
**Alternative Evaluator (AE)**

- AE evaluates all possible alternatives and generates an investment alternatives table with their LCC, system reliability, and service reliability.

- Service reliability needs to be computed based on system reliability and operational data:
  
  \[
  D_{ik} = \left( \frac{T_i}{M_{ik}} \right) NQ_{ik}
  \]

  \[
  M_{ik} = \text{MTBF or MDBF of subsystem } i \text{ with alternative } k
  \]

  \[
  T_i = \text{Operational time or distance of subsystem } i \text{ in a defined period}
  \]

  \[
  Q_{ik} = \text{Average delay (in hours) from a failure of subsystem } i \text{ with alternative } k
  \]

  \[
  N = \text{Average number of online trains}
  \]

  \[
  Q_{ik} \quad \forall i \in I, k \in K
  \]

  Number of failures

  Average train delay
Alternative Evaluator (AE)

- AE evaluates all possible alternatives and generates an **investment alternatives table** with their LCC, system reliability, and service reliability.

- Service reliability needs to be computed based on system reliability and operational data.

\[
D_{ik} = \left( \frac{T_i}{M_{ik}} \right) NQ_{ik}
\]

\[
r_{sys} = \frac{P - \sum_{i \in I} d_i}{P} \times 100\% 
\]

\[
\forall i \in I, k \in K
\]
### Alternative Evaluator (AE)

#### Investment Alternatives

- **Subsystem Alternatives**
  - (i) (k)
  - 1: 29,274, 16.63, 393
  - 2: 39,799, 16.76, 289
  - 3: ...

#### Investment Alternatives Table

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<tr>
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</table>

**Notes:**
- **Delay** ($D_{ik}$) (train-hours)
- **MDBF** ($M_{ik}$) (train-km)
- **LCC** ($C_{ik}$) (billion dollars)
Investment Selector (IS)

- IS identifies the best alternative for every subsystem according to acceptable LCC or service reliability
  - **Minimize Cost Model (MCM):** Minimizing total LCC according to acceptable service reliability
  - **Maximize Reliability Model (MRM):** Maximizing service reliability according to available LCC
Minimize Cost Model (MCM)

Minimize total LCC

\[
\sum_{i \in I} \sum_{k \in K} C_{ik} \delta_{ik} \rightarrow \text{Minimize total LCC}
\]

Subject to:

\[
\sum_{k \in K} \delta_{ik} = 1 \quad \forall i \in I
\]  
\[
d_i = \sum_{k \in K} D_{ik} \delta_{ik} \quad \forall i \in I
\]  
\[
\left( \frac{P - \sum_{i \in I} d_i}{P} \right) \times 100\% \geq R
\]

\[
\delta_{ik} \in \{0,1\} \quad \forall i \in I, k \in K
\]
\[
d_i \geq 0 \quad \forall i \in I
\]

Only one alternative can be chosen for a subsystem

Compute delay for each subsystem

Fulfill the service reliability requirement

<table>
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<tr>
<th>Decision Variables</th>
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<tbody>
<tr>
<td>(\delta_{ik})</td>
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<td>(R)</td>
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whether alternative is selected

delay of subsystem \(i\)

LCC of alternative
delay of alternative
total operational time per year
design service reliability
Maximize Reliability Model (MRM)

\[
\text{Max} \left( P - \frac{\sum_{i \in I} d_i}{P} \right) \times 100\% \quad \rightarrow \text{Maximize service reliability}
\]

\[\sum_{k \in K} \delta_{ik} = 1 \quad \forall i \in I \quad \rightarrow \text{Only one alternative can be chosen for a subsystem}\]

\[d_i = \sum_{k \in K} D_{ik} \delta_{ik} \quad \forall i \in I \quad \rightarrow \text{Compute delay for each subsystem}\]

\[\sum_{i \in I} \sum_{k \in K} C_{ik} \delta_{ik} \leq B \quad \rightarrow \text{Constraint on LCC}\]

\[\delta_{ik} \in \{0,1\} \quad \forall i \in I, k \in K \]

\[d_i \geq 0 \quad \forall i \in I \]

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Two case studies to demonstrate the potential use

- Two case studies with empirical data obtained from a rail system in Taiwan were performed to show the potential use of the proposed method

  - **Case I: New System Design**
    - Designing a new passenger rail system
    - Selecting appropriate alternatives for subsystems according to *design service reliability* → MCM

  - **Case II: Existing System Improvements**
    - Improving the reliability of an existing rail system
    - Subject to constraint on *available increment in LCC* → MRM
Case I: New System Design

- 25-km passenger rail system
- Estimated demand is 140,000 passengers per day
- Six subsystems: train, signal, communication, electricity, station, and track
- Design service reliability (on-time arrival percentage) is from 95% to 99%, with 1% increments

### Subsystem Alternatives

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High design service reliability results in high MDBF and LCC

- MCM efficiently solved this problem by using CPLEX within seconds

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The difference in MDBF among subsystems becomes obvious as service reliability level increases.
> Increase in total LCC from 95% to 97% is modest but very sharp from 97% to 99% because of the nonlinear relationship between cost and reliability.
Case II: Existing System Improvement

- Demand is the same as Case I and service reliability of the existing system is 97% with improvement LCC from 1 ~ 5 billions
- Not all of the subsystems can be easily changed so we consider alternatives for communication, electricity, and track in this case
- MRM efficiently solved this problem by using CPLEX within seconds

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Resulting MDBF from MRM

- Track has the most significant increase in MDBF, followed by communication and electricity.
More LCC have been allocated to electricity, and communication for all scenarios.

Impact on delay from communication and electricity failures is more severe than that from track failures.
Conclusions

- This research develops an **optimization framework** to assist decision makers in optimally allocating service reliability, system reliability, and LCC.

- It is essential to incorporate **service reliability** in rail system design.

- The accuracy of the reliability data obtained from suppliers is a key to successfully determine the optimal investment plan through this process:
  - Test process and data
  - Actual performance data
  - Certificate of conformity

*The proposed tool can help operators maximize their return on investment and provide reliable service to their customers.*
Thank You!

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