Railway Crew Pairing with Weekend Considerations: Modeling and Algorithm

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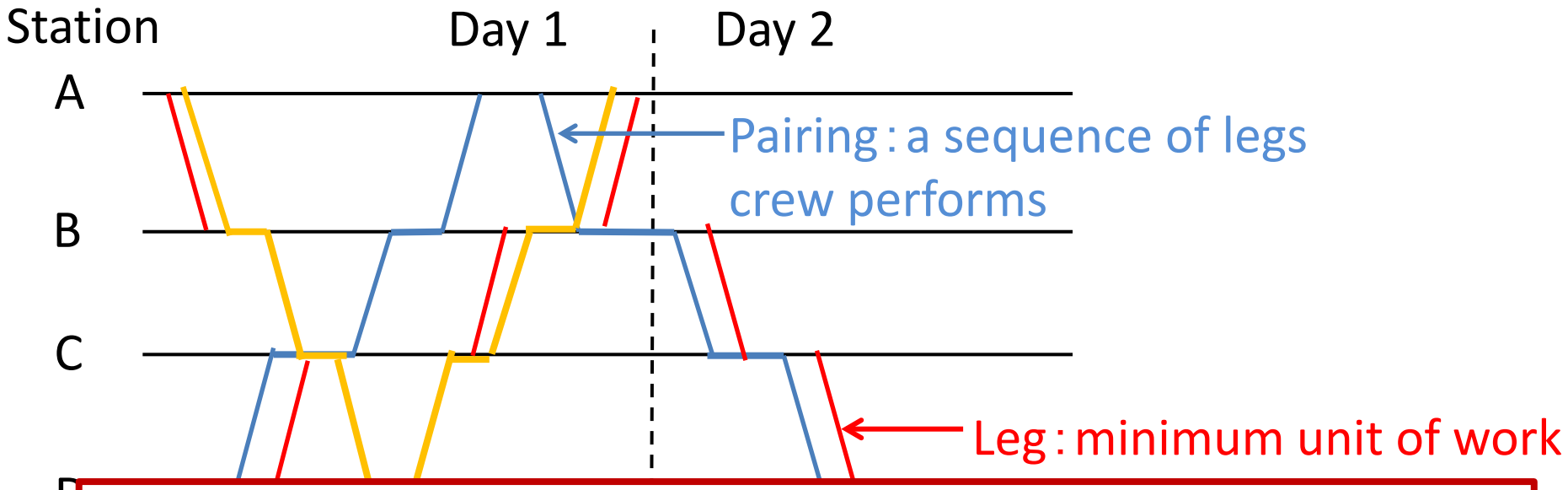
Research Background and Objective

- Background :
  - Crew pairing problem studied extensively in railway OR
  - Column generation established as a standard approach
- Difficulties with the traditional approach :
  - Existence of weekday as well as holiday timetables
  - Overnight pairings may not be used unless existence of different timetables is considered
- Objective :
  - Modeling and algorithm development to generate crew pairings which consider holiday timetable

Crew Pairing Problem



Starting Station (Crew Base) = Ending Station (same Crew Base)

Cr
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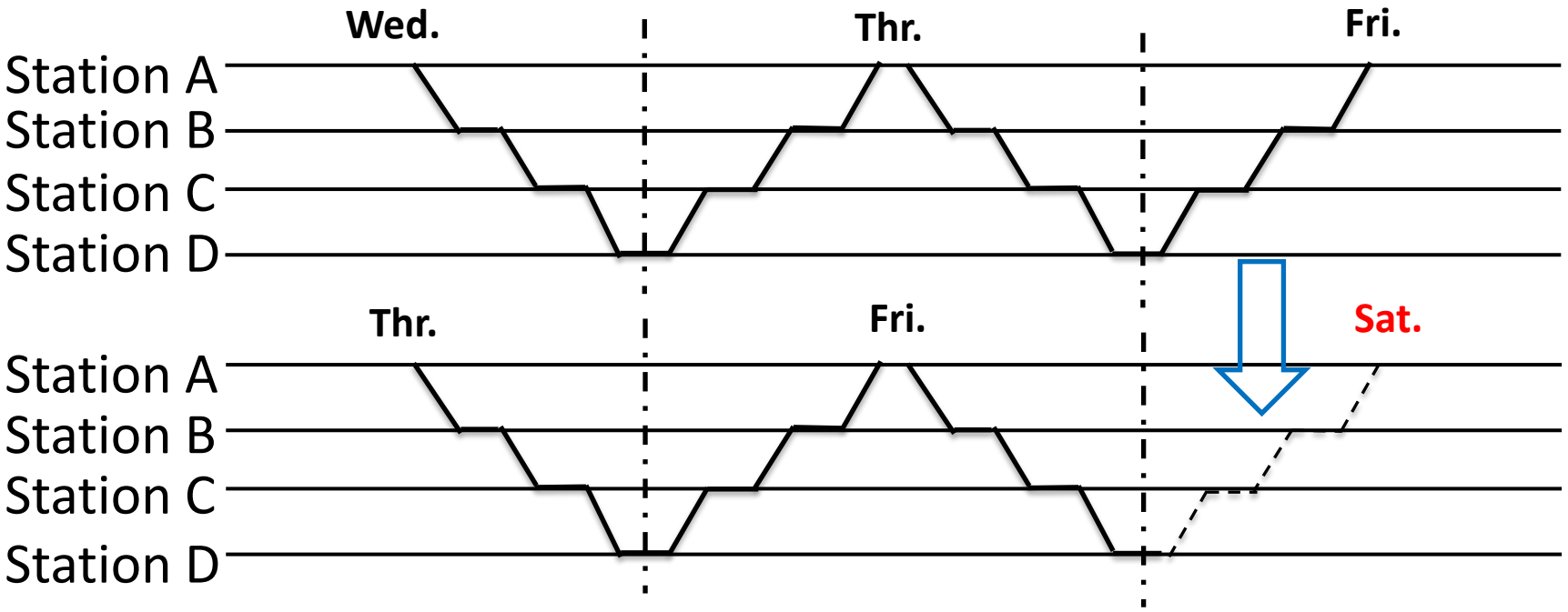
Work Hours
Drive Hours
Rest Hours
Sleep Hours of an Overnight Paring

Ex

Number of Train Legs in a Pairing
Earliest Work Start Time, Latest Work Completion Time, Etc.

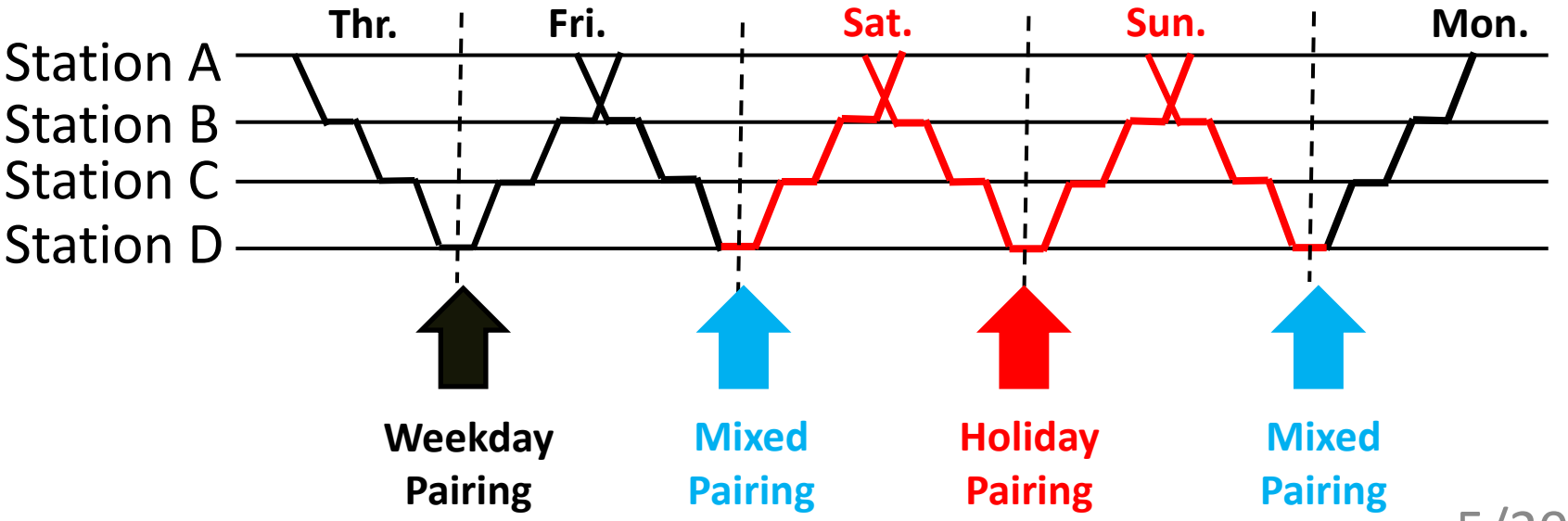
Necessity to Consider Holiday Timetable

- Certain train legs may not exist on holiday



Different Types of Overnight Pairings

- Majority of pairings are overnight in practice
- 3 (or 4) different types of overnight pairings
  - Which timetables are used on 1st and 2nd day?
  - Weekday pairing, Holiday pairing, Mixed pairing



Only a few studies refer to existence of different timetables

[1] Abbink,E., Van`t Wout,J., Huisman,D.(2008): `` Solving large scale crew scheduling problems by using iterative partitioning, " Econometric Institute Report EI2008-03, Erasmus University, Rotterdam.

- Weekly railway crew pairing generation
- Difference of timetables depending on day
- Use of software called TUNRI

[2] Bazargan,M.(2010): Airline Operations and Scheduling, 2nd edition, Ashgate, Surrey, England.

- Airline crew scheduling
- Difference of weekday and holiday timetables
- Modifying weekday pairings on holiday

Two-Stage Approach

1. Construct weekday pairings based on weekday timetable

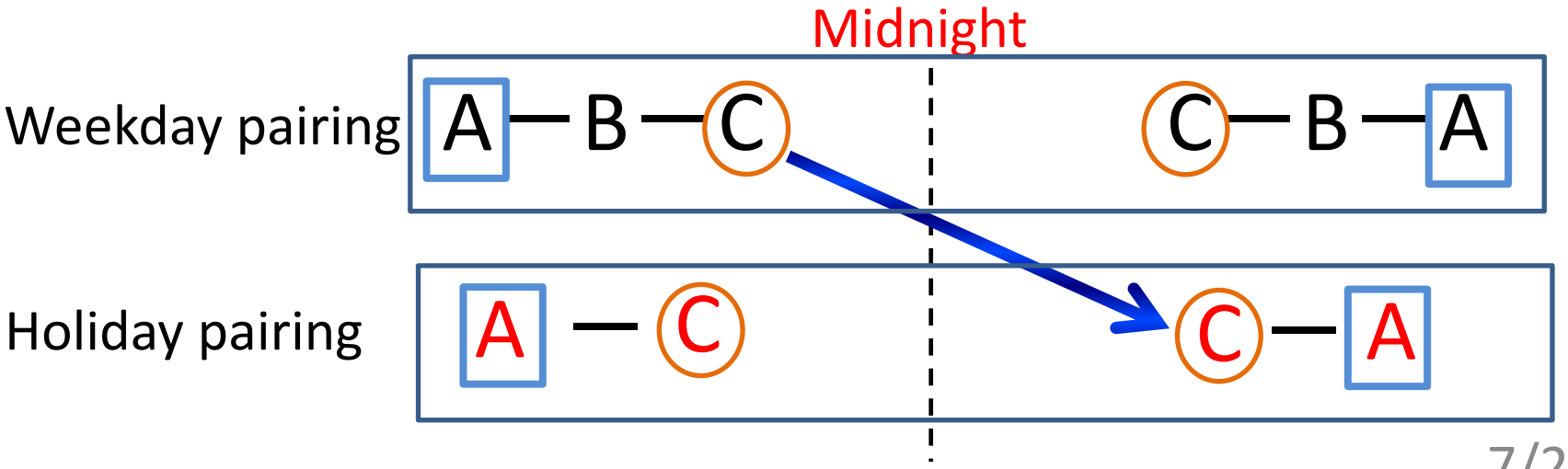
   Use any standard procedure.

2. Construct holiday pairings based on holiday timetable

   Make 1:1 correspondence between a weekday pairing and a holiday pairing.

   – Must satisfy pairing constraints

   – Hope to generate feasible mixed pairings by "concatenation"



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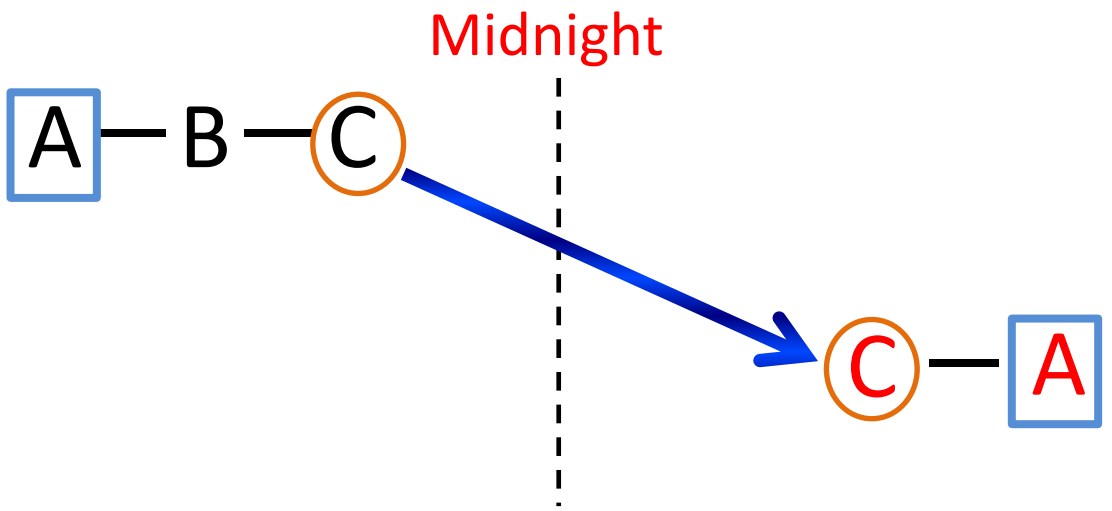
Mixed weekday-holiday pairing



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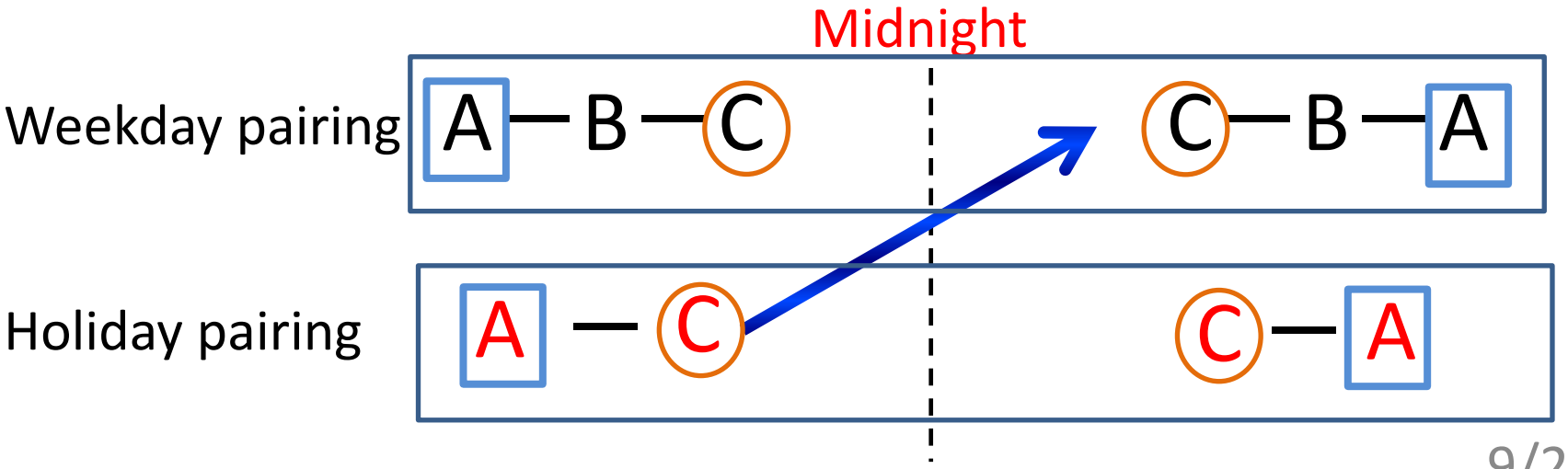

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   Use any standard procedure.

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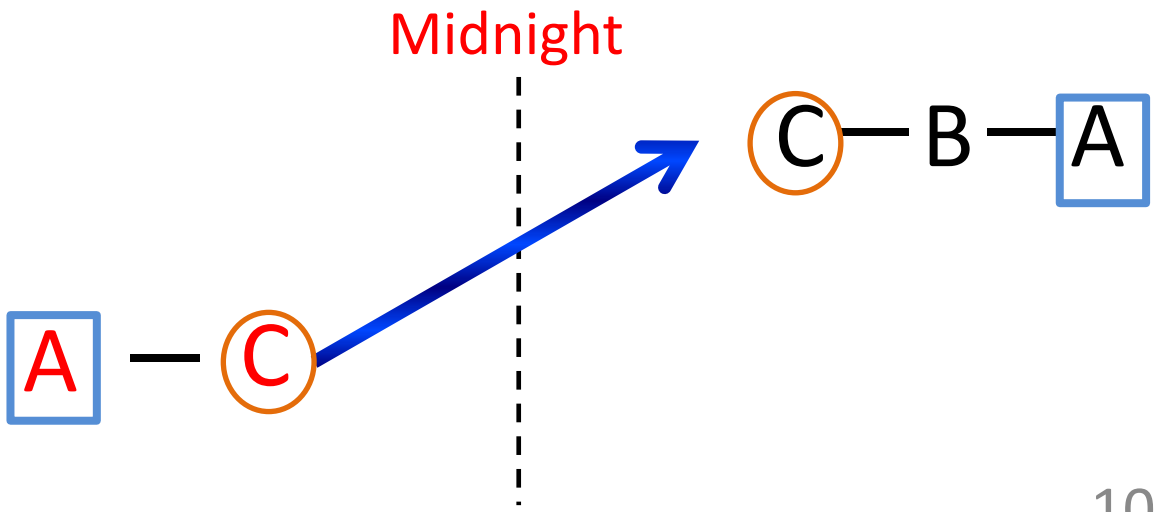
Mixed holiday-weekday pairing



Two-Stage Approach

1. Construct weekday pairings based on weekday timetable

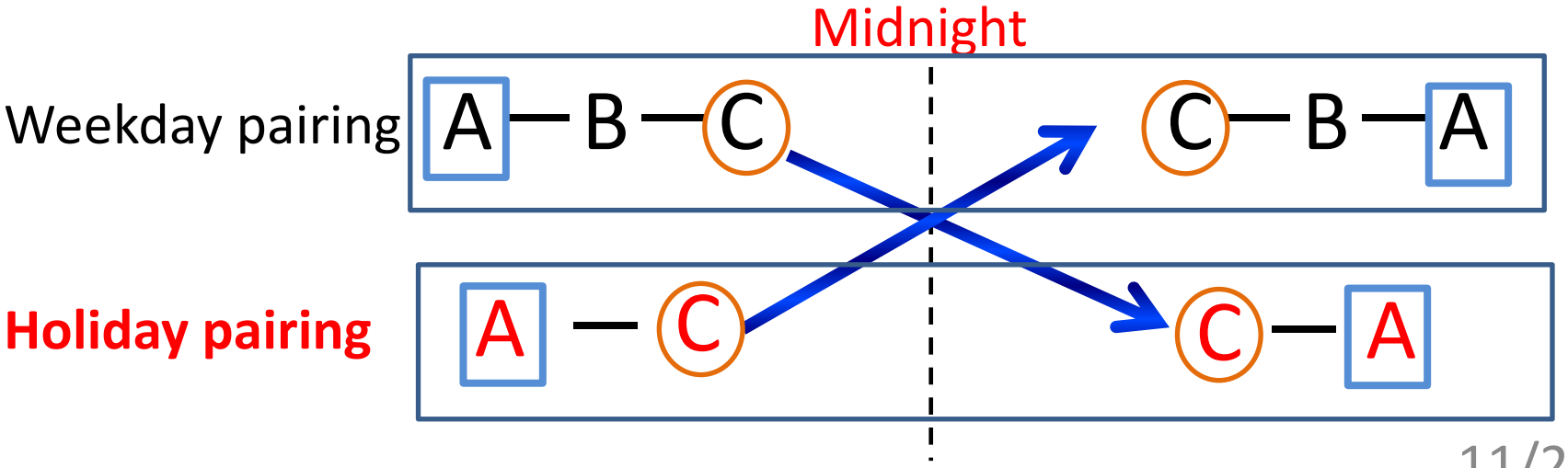
   Use any standard procedure.

2. Construct holiday pairings based on holiday timetable

   Make 1:1 correspondence between a weekday pairing and a holiday pairing.

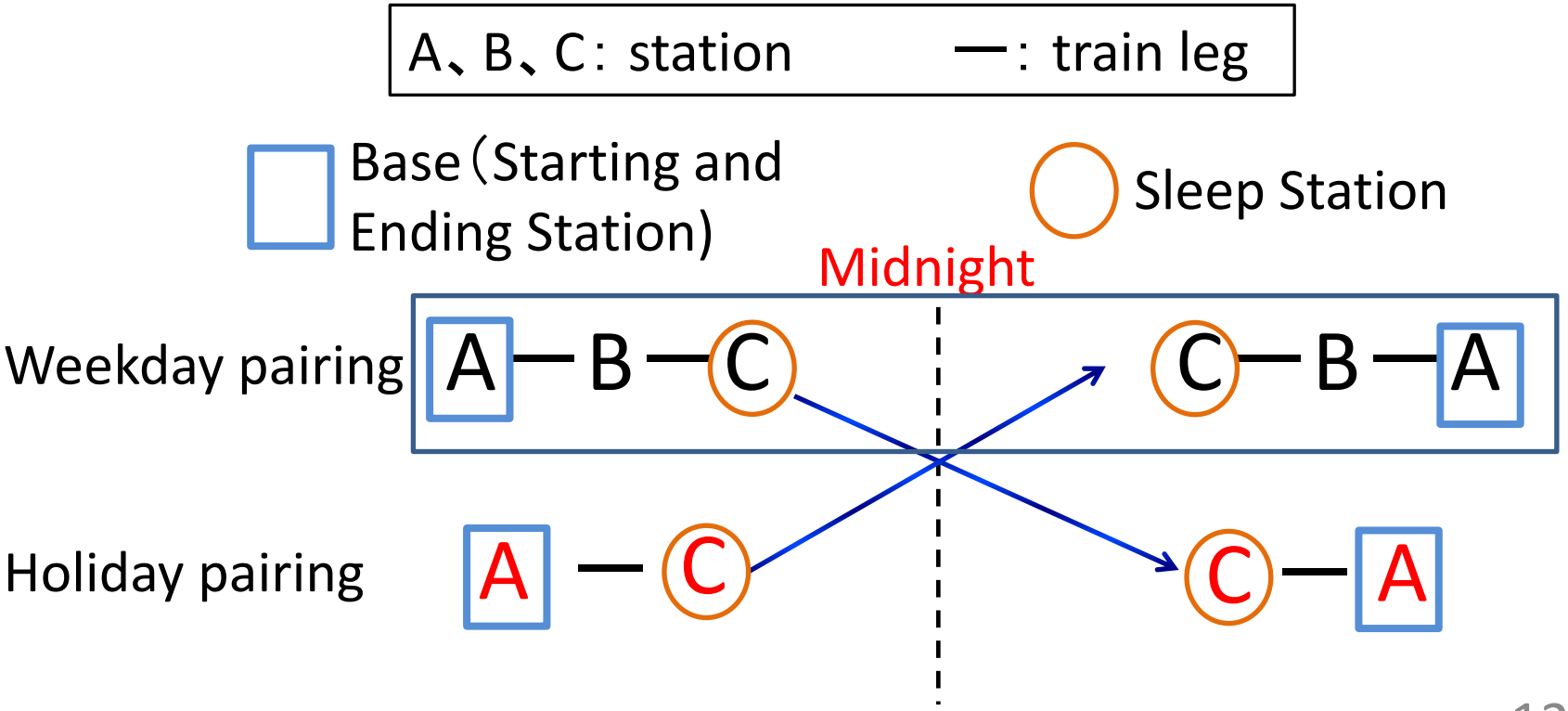
   - Satisfy pairing constraints
   - Also generate feasible mixed pairings by "concatenation"

Holiday Pairing Problem



Crew Base and Sleep Station Must Coincide

- Crew base and sleep station of a holiday pairing must coincide with those of the corresponding weekday pairing.



Conditions to Be Satisfied by Holiday Pairing Problem

(1) The generated holiday pairings satisfy all conditions to be satisfied by pairings.

(2) The concatenated mixed pairings satisfy

- **Crew base constraint**
- **Maximum work hours**
- **Minimum and maximum sleep hours**

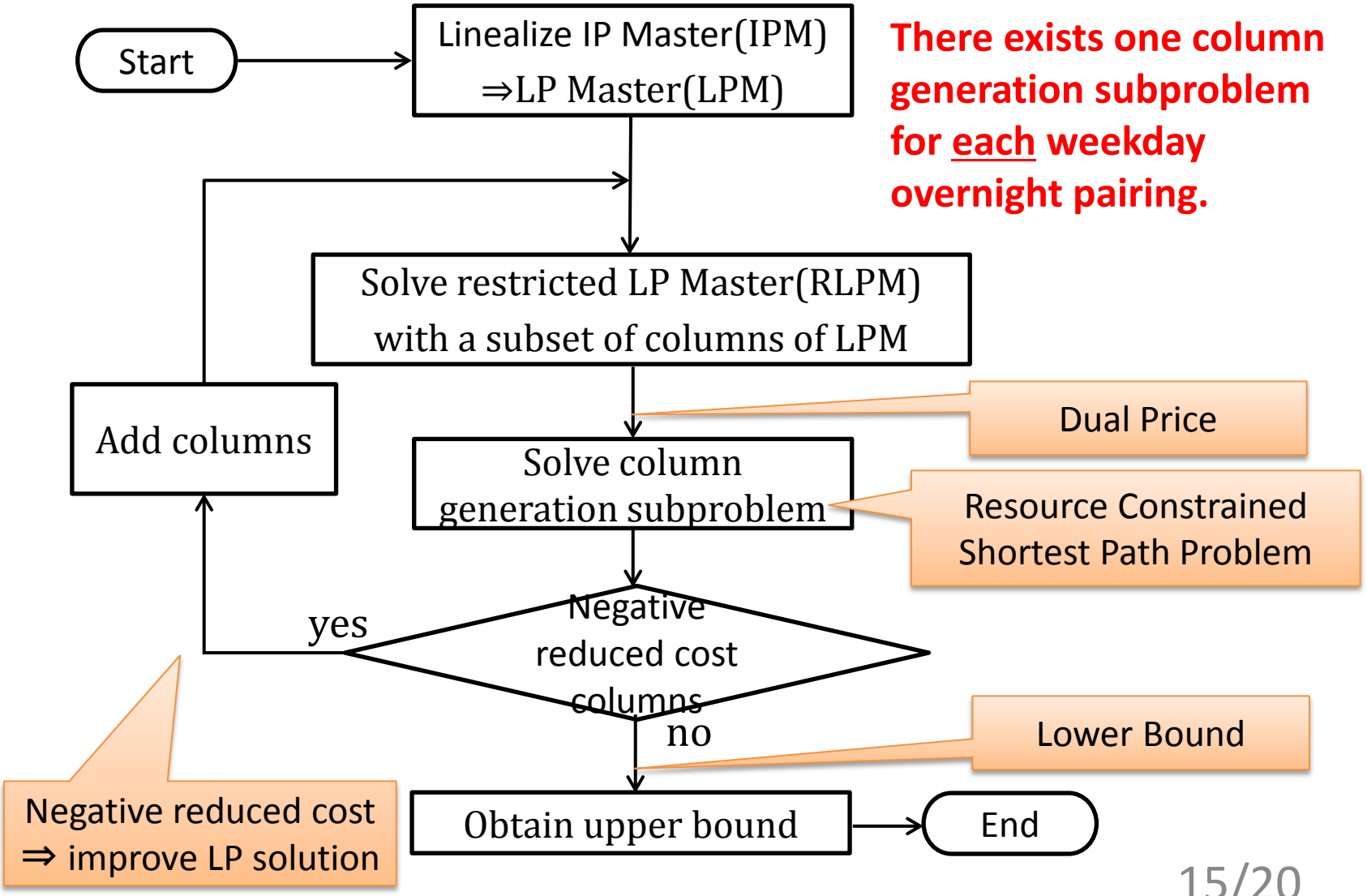
(3) The start time of weekend overnight pairing is the same as that of the corresponding holiday pairing.

We also desire the generated a holiday pairing to be similar to the corresponding weekday pairing.

Similarity of a Holiday Pairing and the Corresponding Weekday Pairing

- Similarity measured by
  - Work hours (penalize more work on holiday)
  - Sleep hours (penalize reduced sleep on holiday)
- Similarity is expressed in the objective function of the holiday pairing problem as a "desirability" measure.

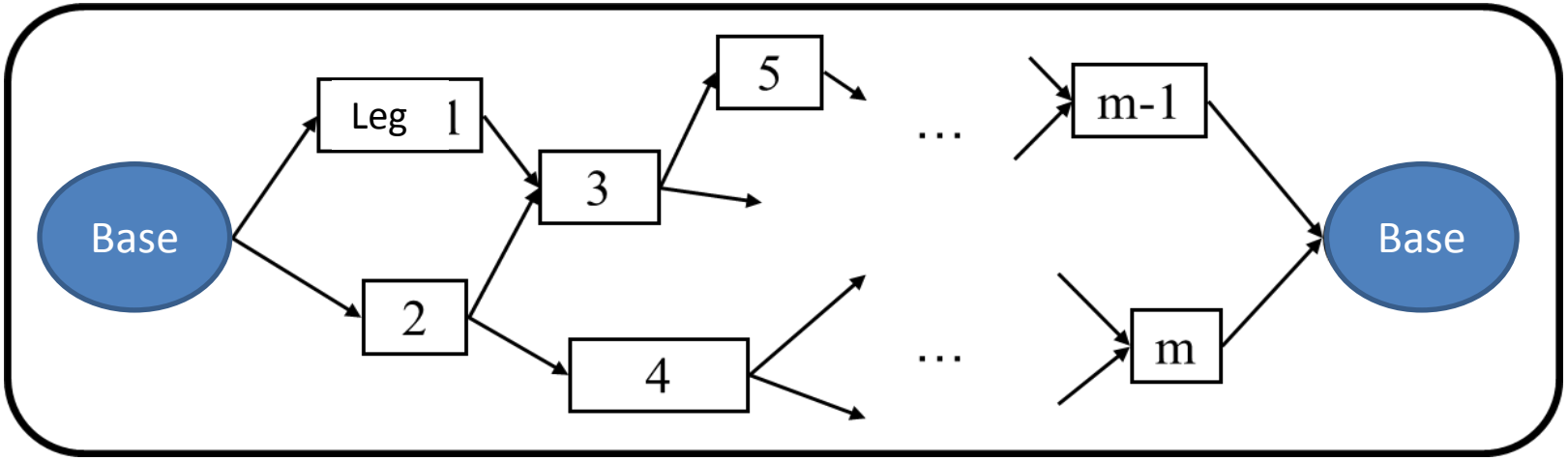
Column Generation Algorithm



Column Generation Subproblem

Resource-Constrained Shortest Path Problem

Network（Node=Leg，Arc＝”Connectability” of legs）



Constraints of Mixed Pairing	How to satisfy constraints?	How similar pairings are?
Max Work Hours	Identical start time of pairings Condition (3)	Cost on arc
Max & Min Sleep Hours	Eliminate certain arcs on the network	Cost on arc

Two Variants of Holiday Pairing Problem

- Two variants considered depending on how one-day pairings are generated

- Variant 1 :
  - Consider correspondence with weekday one-day pairing
  - (One-day pairing) 1:1 correspondence model

- Variant 2 :
  - Do not consider correspondence with weekday one-day pairing; Instead, generate pairings for each crew base
  - (One-day pairing) no correspondence model

Numerical Experiments

Objective

1. Performance evaluation using JR Nara line (Kyoto↔Nara)
   # of weekday／holiday train legs ＝ 174／162
2. Comparisons of 2 variants of holiday pairing generation

Method

- PC: Corei7-3770s, 8GB memory, Windows7 64-bit
- Performed experiments when # of weekday one-day pairings ＝ 1, 2, 3, 4

Holiday Pairing Problem Solved Quickly

Table 1: Comparison of CPU time (sec.)
(# of weekday one-day pairings =2)

	LP Relaxation	Integer Program
Weekday Pairing	32.04	1440.22
Holiday Pairing		
One-day 1:1 correspondence	24.04	34.06
One-day no correspondence	27.55	31.04

“No Correspondence” Model Seems to Yield More Similar Pairings

Table 2：Comparison When # of Weekday One-day Pairings is **2**

	1:1 Corresp.		No Corresp.
Total # Weekday Pairings	22		22
Objective Value （①＋②）	40h 41min		33h 43min
①Diff. of Work Hours	29h 43min	>	23h 52min
②Diff. of Sleep Hours	10h 58min		9h 51min

Table 3：Comparison When # of Weekday One-day Pairings is **3**

	1:1 Corresp.		No Corresp.
Total # Weekday Pairings	23		23
Objective Value （①＋②）	28h 29min		26h 16min
①Diff. of Work Hours	19h 58min	>	17h 22min
②Diff. of Sleep Hours	8h 31min		8h 54min

No correspondence model allows more flexibility.

Larger # of weekday one-day pairings allows more flexibility.

Conclusions

- Succeeded to generate 4 different sets of crew pairings which take holiday timetable into considerations, based on column generation.

- Appeared that better results could be obtained by allowing more room of flexibility in weekday one-day pairings.

References :

[1] Abbink,E., Van`t Wout,J., Huisman,D.: `` Solving large scale crew scheduling problems by using iterative partitioning, " Econometric Institute Report EI2008-03, Erasmus University, Rotterdam, 2008.

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