Temporal Spreading of Alternative Trains in order to Minimise Passenger Travel Time in Practice

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**Business Problem**

**Belgian Infrastructure Management Company: Infrabel:**

Find Timetable that Minimises Expected Passenger Travel Time (includes: ride, dwell, transfer time and primary & secondary delays)

**Note:**

Reduce Expected Passenger Time ⇒ Optimises Robustness

**Fixed:**

Infrastructure, Train Lines, Halting Pattern, Primary Delay Distributions

**Variable:**

Timing: Supplement Times at every Ride, Dwell, Transfer Action, ⇒ variable inter-Train Heading Times ⇒ variable Train Orders

**Specifics:**

One Busy Day, Morning Peak Hour
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**Context:** FAPESP: Two Phased

![Diagram showing the process flow](image)

**Figure:** Two Phased implies Iterations
Graph for Reflowing: add Source & Sink Edges

space increase

time increase
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Timetabling Context

Reflowing & Retiming

Result of Reflowing: Disc Area = Daily Flow
Graph for Retiming: add Knock-On Edges & Cycles
Graph for Retiming: All Constraints

- **primary edges**
  - ride
  - dwell

- **secondary edges**
  - transfer
  - knock on
  - turn around
  - symmetry

- **cycles**
  - \[ \text{sum}_e \text{ in } \text{cycle} = \text{sign}_e \text{ in } \text{cycle} \times (m_e + s_e + (d_e \times T)) = 0 \]

- **Equations**
  - \[ b + m + s = e \]
  - \[ b + m + s + (d \times T) = e \]
  - \[ e = T - b \text{ or } e = T/2 - b \text{ or } ... \]

- **Variables and Constants**
  - \( b, m, s, e, d \)
  - \( m, T \) (integer)
  - \( T \) (period)

Reflowing decides on Rectangle Heights
Retiming (Timetabling) decides on Rectangle Widths
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Timetabling Context

Results without Temporal Spreading

Expected (Non-)Linear Time, as used in Evaluation
"It is a Latin adage that is used either to suggest a hidden motive or to indicate that the party responsible for something may not be who it appears at first to be."
Cui Bono? To Whose Benefit?

- Passengers arriving at station randomly minimise their waiting time before departure
- "inter-departure waiting time"
- "inter-arrival waiting time"
- do benefit:
  - random arrival passengers (fraction $r$)
- do NOT/barely benefit:
  - passengers adapting to train departure schedule (fraction $1 - r$)
## Categories Determining $r$

<table>
<thead>
<tr>
<th>informed</th>
<th>caring</th>
<th>adaptable</th>
<th>adapting</th>
</tr>
</thead>
<tbody>
<tr>
<td>unadaptable</td>
<td>not-caring</td>
<td>not adapting</td>
<td></td>
</tr>
<tr>
<td>uninformed</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>passenger choice model</th>
<th>for dep. time</th>
<th>for dep. over time</th>
<th>for trsf. time</th>
<th>for trsf. over time</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1 - $r$)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Table: Sub-categories of passengers: fraction $(1-r)$ showing passenger choice model behaviour and fraction $r$ showing random arrival behaviour. (dep. = departure, trsf. = transfer)
How Much do they Benefit/Suffer?

[Welding(1957)], [Holroyd and Scraggs(1966)], [Osuna and Newell(1972)]:

\[
E(w) = E(h)/2 \cdot (1 + C_v(h)^2) \tag{1}
\]

\[
C_v(h) = \sigma(h)/\mu(h) \tag{2}
\]

\[
E(h) = \sum_{i=0}^{N-1} p_i \cdot H_i = \sum_{i=0}^{N-1} (H_i/T) \cdot H_i = \sum_{i=0}^{N-1} H_i^2/T \tag{3}
\]

\[
E(f \cdot w) = \frac{f}{2T} \sum_{i=0}^{N-1} H_i^2 \cdot (1 + C_v(h)^2) \tag{4}
\]
ILP Model: Constraints

\( \forall (O, D) : \left\{ \begin{array}{l}
\forall i \in I_N \setminus \{N - 1\} : \; \bar{b}_i \leq \bar{b}_{i+1} \\
\bar{b}_{N-1} \geq \bar{b}_0 
\end{array} \right. \) \hspace{1cm} (5)

\( \forall (O, D) : \forall i \in I_N : \left\{ \begin{array}{l}
\bar{b}_i = \sum_{j \in I_N} p_{i,j} \cdot b_j \\
\forall j \in I_N : p_{i,j} \in \{0, 1\} \\
\sum_{j \in I_N} p_{i,j} = 1 = \sum_{j \in I_N} p_{j,i} 
\end{array} \right. \) \hspace{1cm} (6)

\( \forall (O, D) : \left\{ \begin{array}{l}
\forall i \in I_N \setminus \{N - 1\} : s_i = \bar{b}_{i+1} - \bar{b}_i \\
s_{N-1} = (\bar{b}_0 + T) - \bar{b}_{N-1} \\
\forall i \in I_N : \; 0 \leq s_i \leq T - \delta 
\end{array} \right. \) \hspace{1cm} (7)

\( \forall (i, j) \in I_n : 0 \leq b_i \leq T - \delta \) and so for the non-decreasingly ordered \( \bar{b}_i \) it holds that \( \forall i \in I_N \setminus \{N - 1\} : 0 \leq \bar{b}_{i+1} - \bar{b}_i \leq T - \delta. \)

\( \forall (O, D) : \sum_{i \in I_N} s_i = T \) \hspace{1cm} (8)
ILP Model: Objective Function $=$ How Much Penalty?

Passengers arriving at $O$ and wanting to go to $D$, before having taken first train:

- their expected waiting time for randomly arriving passengers between train $i$ and train $i+1$ is

$$u_i = \int_0^{s_i} (s_i - t) \cdot f \, dt = s_i f \int_0^{s_i} dt - f \int_0^{s_i} t \, dt = f \frac{s_i^2}{2}. \quad (9)$$

- their total expected waiting time for randomly arriving passengers is

$$\forall (O, D) : U = \sum_{i \in I_N} u_i = f / 2 \cdot \sum_{i \in I_N} s_i^2. \quad (10)$$
ILP Model: Piecewise Linearisation

∀(O, D) : ∀i ∈ IN : \begin{cases} 
(s_{i,0}, u_{i,0}) = (0, 0) \\
(s_{i,1}, u_{i,1}) = \left(\frac{T}{N}, \frac{f}{2} \left(\frac{T}{N}\right)^2\right) \\
(s_{i,2}, u_{i,2}) = \left(T, \frac{f}{2} T^2\right)
\end{cases} \tag{11}

∀(O, D) : ∀i ∈ IN : \begin{cases} 
\forall i ∈ IN
\begin{align*}
&u_i \geq u_{i,0} + \frac{u_{i,1} - u_{i,0}}{s_{i,1} - s_{i,0}} \cdot (s_i - s_{i,0}) \\
&= 0 + \frac{f T^2}{2 N^2} (s_i - 0) = \frac{f T}{2 N} s_i \\
&u_i \geq u_{i,1} + \frac{u_{i,2} - u_{i,1}}{s_{i,2} - s_{i,1}} \cdot (s_i - s_{i,1}) \\
&= \frac{f T^2}{2 N^2} + \frac{\frac{f T^2}{2} - \frac{f T}{2 N}}{T - \frac{T}{N}} (s_i - \frac{T}{N}) \\
&= \frac{f T^2}{2 N^2} + \frac{N}{(N-1) \cdot T} \left(\frac{f T^2 N^2 - f T^2}{2 N^2}\right) (s_i - \frac{T}{N}) \\
&= \frac{f T^2}{2 N^2} \left[1 + \frac{N(N+1)}{T} (s_i - \frac{T}{N}) \right]
\end{align*}
\end{cases} \tag{12}
ILP Model: Variable Linearisation

Introduce helper variables $h_{i,j}$, such that

$$\forall (O, D) : \forall i, j \in I_N : h_{i,j} = p_{i,j} \cdot b_j,$$

which is in linearised form:

$$\forall (O, D) : \forall i, j \in I_N : \left\{ \begin{array}{l}
(b_l - b_u)(1 - p_{i,j}) \leq h_{i,j} - b_j \leq (b_u - b_l)(1 - p_{i,j}) \\
b_l \cdot p_{i,j} \leq h_{i,j} \leq b_u \cdot p_{i,j}.
\end{array} \right.$$

So, now

$$\forall (O, D) : \forall i \in I_N : \bar{b}_i = \sum_{j \in I_N} p_{i,j} \cdot b_j$$

can be replaced with:

$$\forall (O, D) : \forall i \in I_N : \bar{b}_i = \sum_{j \in I_N} h_{i,j}.$$
Results Graphical, 26 trains, (Soft, Soft) (O,D)-Spreading
Results Graphical, 26 trains, (Hard, Hard) (O,D)-Spreading
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Results

Results Graphical, 26 trains, (Hard, Soft) (O,D)-Spreading
Results

- 26 trains
  - with transfers
    - soft-soft spreading, 6.55% reduction
    - hard-hard spreading: 3.72% reduction: saves most spreading time, but overstretched, so more ride & dwell time
    - hard-soft spreading: 9.75% reduction

- 200 trains
  - no solution yet
Conclusions

- derived and implemented cost function that measures inter-departure wait time:
  - to evaluate and optimise a schedule on total Excess Journey Time
  - which is addable to other expected passenger time (ride, dwell, transfer, knock-on times)
  - that cannot render the model infeasible
- currently still computationally challenging
Future Work

- try to **control** computation time by:
  - manual i.o. automatic selection of corridors that need spreading
- try to **decrease** computation time by:
  - addition of spreading specific cycles
  - fixing some alternative train orders (breaks 'symmetry', since they are 'the same')
- compare (computation time and solution quality) with classical method of imposing temporal spreading via hard constraints
Questions / Next Steps

- Your questions?
  - here and now, or ...
  - sels.peter@gmail.com
  - www.LogicallyYours.com/research/

- My questions:
  - percentage of non-adapting passenger \( r =?\%
  - tips/tricks to reduce computation time?
