A multi-criteria decision support system for real-time train rescheduling

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Introduction

- Key problem in railway management is the efficient **rescheduling trains** during operation.

- Due to the limited time, train dispatchers may have a limited view on the effects of conflict resolution methods and are not able to compare **alternative solutions**.

- **No clear agreement** in literature on the objective functions to be used

- We aim to develop a **multi-criteria decision support system** to help dispatchers in taking more informed decisions when dealing with real-time disturbance management.
Microscopic Optimization Model

- Based on the Alternative Graph Model. Let $G = (N,F,A)$:
  - $N$ is the set of nodes, each related to the operations in the schedule. To each node $i \in N$ is associated a start time $t_i$ of operation $i$.
  - $F$ is the set of fixed directed arcs that model the sequence of operations to be executed by trains.
  - $A$ is the set of alternative pairs of directed arcs that model the train sequencing decision when a potential conflict arises.

A selection $S$ is a set of alternative arcs, at most one from each pair. A solution is a complete selection $S^c$, where an arc for each alternative pair of $A$ is selected, and it is feasible if the connected graph $(N, F, S^c)$ has no positive length cycles.
The Alternative Graph

\[ X = \left\{ \begin{array}{l}
  t \geq 0 \\
  x \in \{0, 1\}^{|A|} \\
\end{array} \right. 
\]

\[ \begin{align*}
  t_{\sigma(i)} - t_i & \geq w^F_{i\sigma(i)} & \forall (i, \sigma(i)) \in F, \sigma(i) \neq * \\
  t_j - t_i + M (1 - x_{ijhk}) & \geq w^A_{ij} & \forall (i, j), (h, k) \in A \\
  t_k - t_h + M x_{ijhk} & \geq w^A_{hk} & \forall (i, j), (h, k) \in A \\
\end{align*} \]
Single Objective Functions (1)

Maximum Tardiness (MT)

\[
\begin{align*}
\min t_* \\
\text{s.t.} \\
t_* - t_k & \geq -d_k \quad \forall (k, *) \in F \\
\{x, t\} & \in X
\end{align*}
\]

Cumulative Tardiness (CT)

\[
\begin{align*}
\min \sum_{k=1}^{\mid K \mid} z_k \\
\text{s.t.} \\
z_k - t_k & \geq -d_k \quad \forall (k, *) \in F \\
z_k & \geq 0 \quad \forall k \in K \\
\{x, t\} & \in X
\end{align*}
\]

Punctuality (P0)

\[
\begin{align*}
\min \sum_{e=1}^{\mid E \mid} v_e \\
\text{s.t.} \\
Mv_e - t_e & \geq -d_e \quad \forall (e, *) \in F \\
v & \in \{0, 1\}^{\mid E \mid} \\
\{x, t\} & \in X
\end{align*}
\]

Cumulative Tardiness End (CTE)

\[
\begin{align*}
\min \sum_{e=1}^{\mid E \mid} z_e \\
\text{s.t.} \\
z_e - t_e & \geq -d_e \quad \forall (e, *) \in F \\
z_e & \geq 0 \quad \forall e \in E \\
\{x, t\} & \in X
\end{align*}
\]
Single Objective Functions (2)

Priority Cumulative Tardiness End (PCTE)

\[
\min \sum_{e=1}^{|E|} f_e z_e \\
\text{s.t.} \\
z_e - t_e \geq -d_e \quad \forall (e, \ast) \in F \\
z_e \geq 0 \quad \forall e \in E \subset K \\
\{x, t\} \in X
\]

Scheduling Deviation

\[
\min \sum_{k=1}^{|K|} (a z_k - b p_k) + \sum_{r=1}^{|R|} a q_r \\
\text{s.t.} \\
z_k - t_k \geq -d_k \\
z_k \geq 0 \\
p_k - t_k \leq -d_k \\
p_k \leq 0 \\
t_r - q_r \leq w_{sr} \\
q_r \geq 0 \\
\{x, t\} \in X
\]

Priority Cumulative Tardiness End Cost (PCTEC)

\[
\min \sum_{e=1}^{|E|} (f_e z_e + c_e v_e) \\
\text{s.t.} \\
z_e - t_e \geq -d_e \\
z_e \geq 0 \\
M v_e - t_e \geq -d_e - u_e \\
v \in \{0, 1\}^{|E|} \\
\{x, t\} \in X
\]

Total Completion

\[
\min \sum_{e=1}^{|E|} z_e \\
\text{s.t.} \quad z_e - t_e \geq 0 \quad \forall (e, \ast) \in F \\
z_e \geq 0 \quad \forall e \in E \\
\{x, t\} \in X
\]

Travel Time

\[
\min \sum_{g=1}^{|G|} (t^g_l - t^g_f) \\
\text{s.t.} \\
\{x, t\} \in X
\]
Data Envelopment Analysis

- It considers a formulation as a **decision-making unit (DMU)** with multiple inputs and outputs as computation time and performance on all indicators.

- DEA evaluates the **relative efficiency of each solution**.

- Besides the identification of relatively efficient and inefficient DMUs and the efficiency ranking of the formulations under study, DEA helps to identify the sources and the level of inefficiency for each of the considered inputs and outputs.

- This translates into an iterative procedure for the generation of improved formulations via the addition of specific constraints on a number of inefficient performance indicators.
The Decision Support System

**Formulations, Inputs, Outputs**

\[
\begin{array}{cccc}
F_1 & F_2 & F_3 & \ldots & F_q \\
I_1 & I_2 & I_3 & \ldots & I_q \\
O_1 & O_2 & O_3 & \ldots & O_q \\
\end{array}
\]

**Additional Constraints**

**FORMULATION ENHANCEMENT MODULE**

**Efficiency Analysis Results**

**Formulations and Solutions**

\[
\begin{array}{cc}
F_{e1} & S_{e1} \\
F_{e2} & S_{e2} \\
F_{e3} & S_{e3} \\
\ldots & \ldots \\
F_{eq} & S_{eq} \\
\end{array}
\]
The infrastructure considered is a large part of the railway network in the east of the Netherlands, with 1000 block sections and 200 stopping locations.


Optimal MILP solutions are computed via the solver IBM ILOG CPLEX MIP 12.0. The experiments are executed on a workstation Power Mac with processor Intel Xeon E5 quad-core (3.7 GHz), 12 GB of RAM.
## Practical Size Instances

<table>
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<th>Formulation</th>
<th>MT&lt;sup&gt;C&lt;/sup&gt;</th>
<th>CT</th>
<th>CTE</th>
<th>PO&lt;sup&gt;C&lt;/sup&gt;</th>
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<th>SD</th>
<th>TC</th>
<th>TT</th>
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</table>

| **Last Iteration** | | | | | | | | | | | |
| Comp. Time (s) | 3.4 | 11.9 | 8.9 | 5.3 | 9.1 | 8.3 | 9.1 | 7.3 | 14.3 | 2.9 | 3.5 |
| Num. Opt. Sol. | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 |
| Opt. Gap % | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| MT<sup>C</sup> | 220 | 290 | 245 | 530 | 278 | 275 | 287 | 264 | 263 | **313** | 604 |
| CT | 4381 | **2486** | 3032 | 5519 | 3668 | 3696 | 2568 | 3244 | 3229 | 6125 | 9300 |
| CTE | **1512** | 1116 | **954** | 1735 | 1058 | 1058 | 1207 | 1032 | 1032 | **2051** | 2917 |
| PO<sup>C</sup> | 16.5 | 14.5 | 14.5 | **11.7** | 14.6 | 14.6 | 14.7 | 15.0 | 15.0 | **16.5** | 16.5 |
| PCTE | 3.4 | 2.8 | 2.3 | 3.7 | **2.2** | 2.2 | 3.2 | 2.5 | 2.5 | 4.2 | 6.2 |
| PCTEC | 3.4 | 2.8 | 2.3 | 3.7 | 2.2 | **2.2** | 3.2 | 2.5 | 2.5 | 4.2 | 6.2 |
| SD | **191** | 171 | 179 | 203 | 185 | 184 | **170** | 181 | 181 | 208 | 242 |
| TC | 513780 | 513449 | 513320 | 514166 | **513438** | 513606 | **513247** | 513247 | 514350 | 515084 |
| TT | 91377 | 91046 | 90917 | 91763 | 91037 | 91037 | 91203 | 90844 | **90844** | 91947 | 92681 |
| MT<sup>T</sup> | 881 | 917 | 881 | 1011 | 893 | 893 | 917 | 881 | 881 | **872** | 1033 |
| PO<sup>T</sup> | 70 | 70 | 70 | 69 | 70 | 70 | 70 | 70 | 70 | **71** | 66 |
DEA Evaluation

| Formulation | $MT^C$ | CT | CTE | $P0^C$ | PCTE | PCTEC | SD | TC | TT | $MT^T$ | $P0^T$
<table>
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<td>0</td>
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</table>

- $P0^T$ is efficient since the other formulations have a poor performance for it;
- TT has the smallest number of efficient solutions, since competitive solutions are found by the other formulations in a shorter computation time;
- $MT^T$ has a high number of inefficient solutions improved by the iterative procedure.
Conclusion & Ongoing Research

- This paper presents a multi-criteria decision support system to help dispatchers in taking more informed decisions when dealing with real-time traffic disturbances.

- An iterative DEA-based procedure is proposed to establish an efficient-inefficient classification of the solution and to improve inefficient formulations in a case to case basis.

- The proposed multi-criteria decision support system is shown to be able to improve the inefficient formulations, and to deliver of a pool of improved formulations and their solutions in a short computation time for practical-size instances.

- The procedure converges after a limited number of iterations and returns a set of efficient formulations and their best solutions. The final selection of the train schedule to be implemented is left to the dispatchers.

- On-going research is dedicated to a more comprehensive evaluation of alternative formulations and DEA classifications.