Train delay evolution as a stochastic process

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Outline

1. Introduction
2. Train delay evolution as a stochastic process
3. Computational experiments
4. Current research
5. Conclusions
1 Introduction

2 Train delay evolution as a stochastic process

3 Computational experiments

4 Current research

5 Conclusions
Uncertainty in railway traffic

Railway traffic typically operates according to a timetable, however...

Source: D’Ariano, PhD thesis

Train delay evolution as a stochastic process

Source: D’Ariano, PhD thesis
Tactical level:

- Stochastic timetable planning
- Running and dwell times are attributed with probability distributions fitted from the historical data

Source: Medeossi et al (2011) JRTPM
Operational level:

- Predictive traffic control, delay management, passenger information
- Prediction in real time based on the available information
- Deterministic running and dwell times
Motivation

- Current practice is based on ‘parallel shift’ - simple extrapolation of the current delays as the expected arrival delays
- Deterministic models require frequent updates of train positions and detailed data
- Stochastic models are based on fixed distributions and do not exploit information available in real time
The dynamics of a train delay over time and space is presented as a stochastic process that describes the evolution of the time-dependent random variable.

Probability distribution of an arrival delay in a station changes over time in discrete steps as more information becomes available.
1. Introduction

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Methodological framework

Traffic model

- A train run as a sequence of discrete events that model arrivals and departures events
- The events are connected by the corresponding running and dwell processes.
- The events occur in a fixed sequence $j \rightarrow k$
- Interaction between trains is not included in the model

Station 1  Station 2  Station 3  Station N

<table>
<thead>
<tr>
<th>dep</th>
<th>arr</th>
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Train delay evolution over a sequence of events represented as a stochastic process

Delay is a time-dependent random variable $X_i = X_1, X_2, \ldots, X_n$

Each random variable $X_i$ represents a delay of event $i$, where $i = 1, 2, \ldots, n$

**Markov property:** delay of a future event can be fully predicted based on the present delay

$$P\{X_{i+1} \mid X_i = x_i, X_{i-1} = x_{i-1}, X_{i-2} = x_{i-2}, \ldots\} = P\{X_{i+1} \mid X_i = x_i\}. \quad (1)$$

Probability of transition form state $x_i$ to state $x_{i+1}$, $\forall i \in 1, \ldots n - 1$ is given by:

$$P\{X_{i+1} = x_{i+1} \mid X_i = x_i\} = p_{i,i+1}. \quad (2)$$
Properties of the Markov process model

State-space definition

- $S_1 = \{ s_1 = [a_1, a_2], s_2 = (a_2, a_3), s_3 = (a_4, a_5) \}$
- $S_2 = \{ a_1, \ldots, -2, -1, 0, 1, 2, \ldots, a_2 \}$
Properties of the Markov process model

Time-variant, non-stationary process

- Delay jumps may have a different probability distributions for running and dwell processes
- Some processes are scheduled with more time reserves which may have a significant impact on the corresponding delay jump
- We model train delay evolution as a non-stationary Markov process

\[ P_{i,i+1} = \begin{pmatrix}
  p_{1,1}(i) & p_{1,2}(i) & \cdots & p_{1,m}(i) \\
  p_{2,1}(i) & p_{2,2}(i) & \cdots & p_{2,m}(i) \\
  \vdots & \vdots & \ddots & \vdots \\
  p_{m,1}(i) & p_{m,2}(i) & \cdots & p_{m,m}(i)
\end{pmatrix} \]  

where \( i = 1, \ldots, n \) and \( m = |S| \)
Given the initial value of the first random variable in the sequence the remaining variables in the sequence can be computed recursively.

Step 1: conditional probability distribution given the transition matrix and the current state of the system:

\[ P[X_{i+1} \mid X_i = j] = x_i \cdot P_{i,i+1} \]  \hspace{1cm} (4)

Step 2: given the conditional probability distribution, the value of the random variable \( X_{i+1} \) can be computed using the Monte-Carlo sampling.
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Data from the high-speed line between Beijing and Shanghai
Data is from the northern part - 5 stations
58 G (300 km/h) trains and 12 D (250 km/h) trains daily per direction
Data between the 1\textsuperscript{st} of December, 2013 and the 4\textsuperscript{th} of March 2014
Only planned and realised time for each departure, arrival and through event (no signal or track data) rounded to full minutes
Test set contains 20\% of randomly selected train runs
Trains are allowed to depart up to 5 minutes before their scheduled departure time.
Analysis performed for each train line (stopping pattern) separately

$S_1$ state-space definition
- if delay $\leq 0 \rightarrow$ delay = ‘early’ (89% of data)
- if $0 <$ delay $\leq 5 \rightarrow$ delay = ‘small’ (6%)
- if delay $> 5 \rightarrow$ delay = ‘large’ (5%)

$S_2$ state-space definition: delays [-5,5] min

Separate transition matrices are computed for each state-space
### Experimental setup

Example of transition matrices

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<td></td>
<td></td>
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<td>0.02</td>
<td>0.01</td>
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<td>0.45</td>
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<tr>
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<td><strong>D Tianjin - A Cangzhou</strong></td>
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<tr>
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Results

Example of delay evolution

Beijing departure  Tianjin arrival  Tianjin departure  Changzhou arrival

Event 1

Event 2

Event 3

Event 4

Event 5
Results

Aggregated results
1. Introduction

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Explicit modelling of the interdependence between trains that:
- Share the same infrastructure
- Have a scheduled passenger transfer, rolling-stock or crew connection

Approach based on Bayesian belief networks
Stochastic modelling of train interactions

Further steps

1. Get and process the data
2. Determine the network structure from the data
3. Compute conditional probabilities
4. Consider different model structures: train-based, infrastructure-based, temporal dynamics, time dependence, network size

\[ X_1^{(1)} \rightarrow X_1^{(2)} \rightarrow \cdots \rightarrow X_1^{(t)} \rightarrow \cdots \rightarrow X_1^{(n)} \]

\[ X_2^{(1)} \rightarrow X_2^{(2)} \rightarrow \cdots \rightarrow X_2^{(t)} \rightarrow \cdots \rightarrow X_2^{(n)} \]

\[ X_3^{(1)} \rightarrow X_3^{(2)} \rightarrow \cdots \rightarrow X_3^{(t)} \rightarrow \cdots \rightarrow X_3^{(n)} \]
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Analysis of stochastic dynamics of train delays

The goal was to determine the impact of real-time information on reducing uncertainty

Reliability of prediction increased by 71% compared to the static case with offline computed distributions

Future work on modelling the dynamic interrelation of delays in a closed form

It is preferable to evaluate the approach on other case studies

Potential applications include integration with online traffic control models, online delay management and passenger information systems
Thank you for your attention